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Differential rotation in Jupiter: A comparison of methods

J. Wisdom [a,*], W.B. Hubbard [b]

[a] Department of Earth, Atmospheric, and Planetary Sciences, Massachusetts Institute of Technology, Cambridge, MA 02139, United States
[b] Lunar and Planetary Laboratory, University of Arizona, Tucson, AZ 85721, United States

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ABSTRACT

Whether Jupiter rotates as a solid body or has some element of differential rotation along concentric cylinders is unknown. But Jupiter's zonal wind is not north/south symmetric so at most some average of the north/south zonal winds could be an expression of cylinders. Here we explore the signature in the gravitational moments of such a smooth differential rotation. We carry out this investigation with two general methods for solving for the interior structure of a differentially rotating planet: the CMS method of Hubbard (Hubbard, W.B. [2013]. Astrophys. J. 768, 1–8) and the CLC method of Wisdom (Wisdom, J. [1996]. Non-Perturbative Hydrostatic Equilibrium. http://web.mit.edu/wisdom/www/interior.pdf). The two methods are in remarkable agreement. We find that for smooth differential rotation the moments do not level off as they do for strong differential rotation.

1. Introduction

Whether Jupiter rotates as a solid body or has some element of differential rotation along concentric cylinders is unknown. The observed zonal wind profile may extend all the way through the planet (Busse, 1976) or be surficial (Williams, 1978). It may also be that the zonal wind profile extends to great depth but not all the way through the planet (Kaspi et al., 2010). If the observed zonal wind profile extends to great depth or goes all the way through the planet, then it is expected that there will be a distinctive leveling off of the high order gravitational moments (Hubbard, 1999). The level at which the moments level off will be indicative of the depth of the zonal winds (Kaspi et al., 2010). But Jupiter's zonal wind profile is not north/south symmetric, so the detailed zonal winds cannot be an expression of cylinders that pass though the whole planet. At most some average of the north/south zonal winds could be the expression of cylinders. Thus it is possible that there is a smooth differential rotation on cylinders that extends throughout the planet with a surficial zonal wind extending to some depth that completes all the wiggles on top of this smooth trend. If the surficial zonal wind does not penetrate deeply then the moments will not level off, but nevertheless there may still be differential rotation on cylinders. Here we explore the behavior of the gravitational moments for such a smooth differential rotation, and compare this behavior to that of a strongly differentially rotating planet.

We carry out this investigation with two general methods for solving for the interior structure of a differentially rotating planet, given a barotropic equation of state. We use the method of Wisdom (1996) and the method of Hubbard (2013). The unpublished work of Wisdom (1996) is attached here as Supplementary material. We refer to the method of Wisdom (1996) as the "consistent level curve" (CLC) method, and the method of Hubbard (2013) as the "concentric Maclaurin spheroid" (CMS) method. Both methods are extended here to handle differential rotation on cylinders.

When computing the external gravity field of a (differentially or uniformly) rotating liquid planet, the principal task is to achieve a self-consistent solution for the interior structure in which the so-called level surfaces of constant density, pressure, and total potential coincide to a specified precision. Because Jupiter rotates rapidly, with rotational potential terms $\sim10^{-1}$ of the gravitational potential, a fully non-perturbative theory is required to achieve self-consistency to considerably better than, say, $\sim10^{-9}$. The key to achieving such precision is to very accurately determine the shape of interior level surfaces. The CLC method does so by precisely solving for a smooth bivariate "shape function" within the planet (see Section 3). In contrast, the CMS method precisely calculates the shape of a finite number of interfaces between uniform-density spheroids.

2. Models

Here we model Jupiter as an $n = 1$ polytrope. For a polytropic equation of state $p = K\rho^{1+1/n}$, with $n = 1$, the pressure p is propor-

* Corresponding author.

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Table 1
Model DR2 coefficients.

n	A_n
2	$-5.8954166431924311100 \times 10^{-2}$
4	$4.6123389620922838894 \times 10^{-1}$
6	$-1.0426642770099761037 \times 10^{+0}$
8	$8.3030636453130002295 \times 10^{-1}$

Table 2
Model DR3 coefficients.

n	A_n
2	$-5.4553556626624011283 \times 10^{-1}$
4	$2.0659730767589977063 \times 10^{+1}$
6	$-2.9751201811994826585 \times 10^{+2}$
8	$2.1388899216795743996 \times 10^{+3}$
10	$-8.7087211563139935606 \times 10^{+3}$
12	$2.1342828803262666042 \times 10^{+4}$
14	$-3.2058440847282883624 \times 10^{+4}$
16	$2.8850390979208579665 \times 10^{+4}$
18	$-1.4264411879490267893 \times 10^{+4}$
20	$2.9770389071934714593 \times 10^{+3}$

tional to the square of the density ρ. This gives a good first approximation to the interior of Jupiter (Hubbard, 1975, 1982). A non-dimensional measure of the relative strength of the centrifugal force to the gravitational force is

$$q_e = (\Omega^2 R_e)/(GM/R_e^2) = \Omega^2 R_e^3/(GM). \quad (1)$$

The basic rotation rate Ω corresponds to a period of $9^h55^m29.7^s$, the equatorial radius R_e is taken to be 71,492 km, and $GM = 126686536.1\ \mathrm{km}^3/\mathrm{s}^2$ (Hubbard, 1982). We take q_e for Jupiter to be 0.089195487 (exactly). This completely specifies the model; the task is to compute the consequent structure and the exterior gravitational moments.

An advantage of the $n = 1$ polytrope is that the corresponding hydrostatic structure can additionally be solved by an independent method involving spherical Bessel functions (Hubbard, 1975, 1999). The details of the Bessel function method without differential rotation were presented in Wisdom (1996). Extension of the method to include differential rotation on cylinders was outlined by Hubbard (1999).

The rotational potential is $Q(c) = \int_0^c c'\Omega^2(c')dc'$; the potential is a function of the perpendicular distance c from the rotation axis. Let $Q = Q_0 + \Delta Q$, where $Q_0(c) = \frac{1}{2}c^2\Omega_0^2$ is the centrifugal potential corresponding to the basic rotation rate Ω_0, and

$$\Delta Q(c) = \sum_{j=1}^{n} A_{2j}(c/R_e)^{2j}. \quad (2)$$

We study three different specifications of the coefficients A_{2j}. For model DR0, all the A_i are zero—the body rotates uniformly. For model DR1, a model displaying weak differential rotation, the constants A_{2j} are taken from Hubbard (1982): $A_2 = 0.017828$, $A_4 = -0.209508$, $A_6 = 0.518688$, and $A_8 = -0.228979$. The units

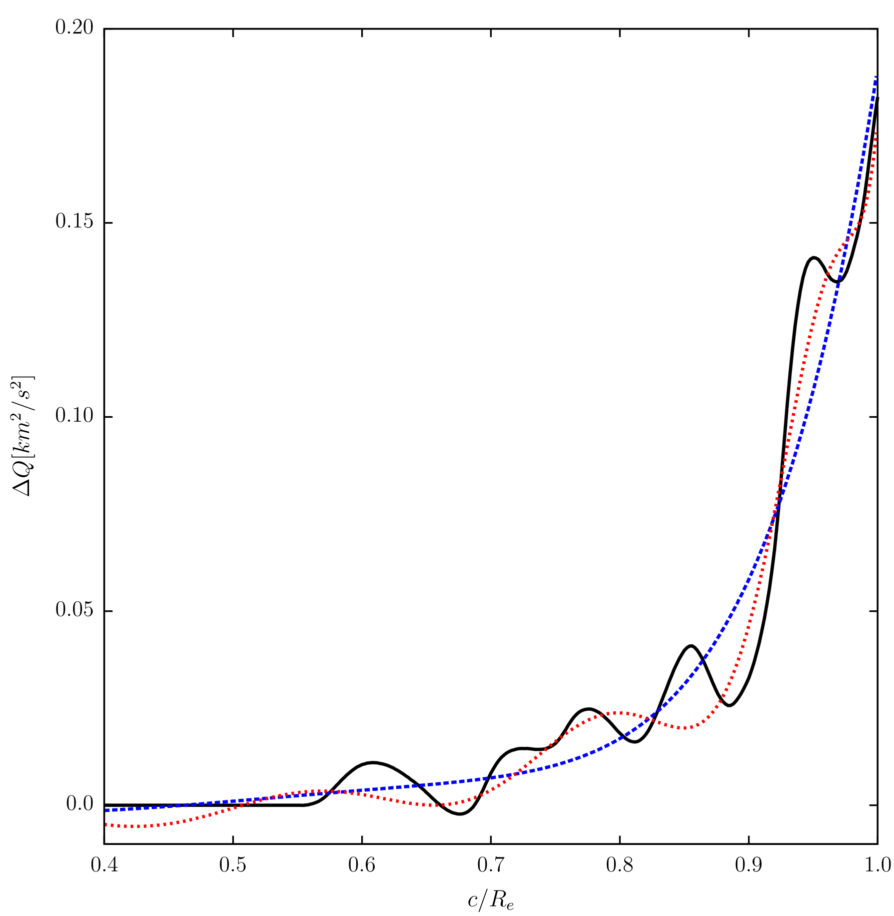

Fig. 1. The effective potential of the zonal winds versus the distance from the rotation axis. The solid black curve is obtained by integrating the northern hemisphere winds of Jupiter as observed by Cassini. The dashed blue curve is the weak differential rotation model (model DR2); the dotted red curve is the strong differential rotation model (model DR3). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)