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Near-equilibrium dumb-bell-shaped figures for cohesionless small bodies

Pascal Descamps *

Institut de Mécanique Céleste et de Calcul des Éphémérides, Observatoire de Paris, UMR 8028 CNRS, 77 av. Denfert-Rochereau, 75014 Paris, France

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ABSTRACT

In a previous paper (Descamps, P. [2015]. Icarus 245, 64–79), we developed a specific method aimed to retrieve the main physical characteristics (shape, density, surface scattering properties) of highly elongated bodies from their rotational lightcurves through the use of dumb-bell-shaped equilibrium figures. The present work is a test of this method. For that purpose we introduce near-equilibrium dumb-bell-shaped figures which are base dumb-bell equilibrium shapes modulated by lognormal statistics. Such synthetic irregular models are used to generate lightcurves from which our method is successfully applied. Shape statistical parameters of such near-equilibrium dumb-bell-shaped objects are in good agreement with those calculated for example for the Asteroid (216) Kleopatra from its dog-bone radar model. It may suggest that such bilobed and elongated asteroids can be approached by equilibrium figures perturbed be the interplay with a substantial internal friction modeled by a Gaussian random sphere.

1. Introduction

In an earlier work, dumb-bell-shaped hydrostatic equilibrium figures were proposed to describe shapes of contact-binary minor planets with a bimodal appearance (Descamps, 2015). However, small bodies in the Solar System are quite obviously not liquid but exhibit many pieces of evidence that most of them are "piles of rubble" (Davis et al., 1979), i.e. loosely consolidated aggregates of collisional fragmented material with zero tensile strength held together by mutual gravitational forces (see for example the review of Scheeres et al., 2010). One of the most convincing evidence results from high porosities measured in asteroids since the discovery of asteroid satellites which allowed density to be estimated. Significant macro-porosity sketches rubble pile models made of grains or boulders resting on each other with large voids. The fluid approach can give only overall shapes consistent with their angular momentum of rotation. Rubble piles which are much weaker than coherent structures are able to withstand shear strength due to their internal friction. This allows a much wider range of possible shapes not necessarily close to equilibrium shapes and thereby topography out of hydrostatic equilibrium can be maintained.

In the present paper, near-equilibrium dumb-bell shapes are introduced as a combination of dumb-bell-shaped equilibrium figures with a Gaussian random sphere which is used to simulate the departures of real shapes from pure equilibrium figures. Although we do not know the true shape statistics of small bodies, the Gaussian hypothesis for the logarithmic radius of the perturbing sphere allows to simulate irregular shapes fully characterized by only a few statistical parameters. This paper aims at testing the reliability of our fitting and modeling protocol by dumb-bell-shaped equilibrium figures (Descamps, 2015) through simulations of noisy rotational lightcurves of pseudo-equilibrium dumb-bell shapes.

2. Near-equilibrium dumb-bell-shaped figures

2.1. Gaussian random sphere

Muinonen (1998) first used lognormal statistics to modeling the irregular shapes of asteroids and cometary nuclei through the so-called *Gaussian random sphere*. The Gaussian sphere is fully described by only three statistical parameters: the mean radius a, the relative standard deviation σ and the correlation angle Γ. In the limit of small standard deviations of radius and small correlation angles, the lognormal statistics reduces to the Gaussian statistics. A suitable covariance function of the logarithmic radius was devised for the generation of Gaussian spheres that closely resemble the shapes observed for asteroids. In spherical coordinates, the

* Fax: +33 (0)146332834.
E-mail address: descamps@imcce.fr

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Table 1
Statistical properties of some rubble-pile asteroids (Section 2.2).

Object	$\tilde{\sigma}$	$\tilde{\rho}$	$\tilde{\Phi}$ (°)	$\tilde{\Gamma}$ (°)
22 Kalliope	0.15	0.30	16.67	28.64
45 Eugenia	0.18	0.35	19.46	20.04
87 Sylvia	0.12	0.23	12.92	29.32
107 Camilla	0.12	0.23	13.07	29.32
121 Hermione	0.29	0.59	30.61	28.22
130 Elektra	0.12	0.23	12.71	29.46
216 Kleopatra	0.62	0.75	36.77	44.67
1999 KW4a	0.07	0.23	13.13	16.97
1999 KW4b	0.15	0.28	15.47	30.58
2867 Steins	0.15	0.34	18.59	25.29
4769 Castalia	0.27	0.53	28.74	28.14
25143 Itokawa	0.31	0.47	25.34	37.03

radius of a Gaussian random sphere, $r(\theta, \varphi)$ with respect to its center of mass can be fully defined by the mean radius and the covariance function of the logarithmic radius $s(\theta, \varphi)$.

$$r(\theta,\varphi) = \frac{a\exp(s(\theta,\varphi))}{\sqrt{1+\sigma^2}}$$
$$s(\theta,\varphi) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} s_{lm} Y_{lm}(\theta,\varphi) \tag{1}$$

$s(\theta,\varphi)$ is the logarithmic radial distance, a and σ are respectively the mean radius and standard deviation for the radial distance $Y_{lm}(\theta,\varphi)$s represents orthonormal spherical harmonics. The value of radial standard deviation σ quantizes the irregularity in shape relative to a sphere ($\sigma = 0$ for spherical bodies). The s_{lm}s are random variables which obey normal statistics with zero means and variances depending on the shape statistics specified by the so-called covariance function of logradius. If we denote the angular distance between two directions (θ_1, φ_1) and (θ_2, φ_2) by γ, the covariance function $\Sigma_s(\gamma)$ is related to the correlation function $C_s(\gamma)$ by $\Sigma_s(\gamma) = \beta^2 C_s(\gamma)$, where β is the standard deviation of the logradius. The variances σ^2 and β^2 are interrelated through $\sigma^2 = \exp(\beta^2) - 1$. The correlation function is generally described by a series expansion of Legendre polynomials (including lower and upper bounds for the degree), the coefficients of the Legendre polynomials C_l follow a power-law dependence $C_l \propto l^{-\nu}$. It appears to be a true law, with $\nu \sim 4$, for overall shapes of asteroids (Muinonen and Lagerros, 1998) and even for Saharan dust particles (Nousiainen et al., 2003). However, in the present work we make use of the Gaussian correlation function, the correlation between two radii over solid angle γ is then given by

$$C_s(\gamma) = \exp\left(-\frac{1}{2}\frac{\sin^2\gamma/2}{\sin^2\Gamma/2}\right) \tag{2}$$

where Γ is the correlation angle of the Gaussian sphere, defined as the angular displacement over which the correlation drops to $1/\sqrt{e}$. A small correlation angle leads to increase short-distance fluctuations on the body surface (higher number of "valleys" and "hills").

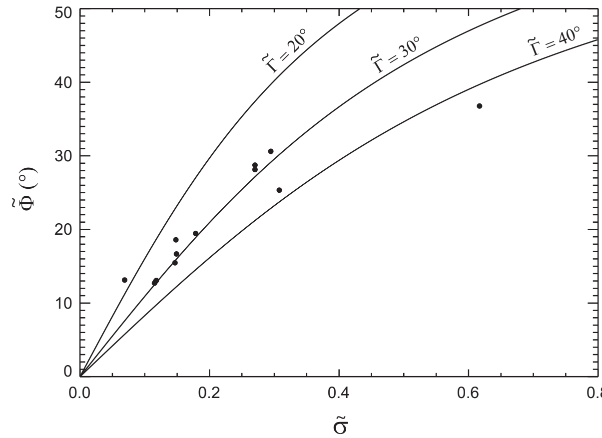

Fig. 1. The standard deviation of slope angle $\tilde{\Phi}$ against the relative standard deviation of the radius $\tilde{\sigma}$ and the correlation angle $\tilde{\Gamma}$ for the 12 individual shapes of rubble-pile asteroids listed in Table 1. Iso-gamma lines are drawn for $\tilde{\Gamma} = 20°$, 30° and 40°. Most of bodies have a correlation angle close to $\tilde{\Gamma} = 30°$ and a standard deviation of the radius ranging from 0.1 to 0.3.

This choice is made suitable from the fact that the elongation and flatness of the figures are already included in the base dumb-bell figure (see Section 2.3). Such a correlation function was also used for the same reason for the natural extension of the Gaussian random sphere in the form of the Gaussian random ellipsoid (Muinonen and Pieniluoma, 2011). Further details regarding the description and generation of Gaussian random spheres are given in Muinonen (1998) and Muinonen and Lagerros (1998).

2.2. Statistics of some irregular shapes of small bodies

The inverse problem of determining the statistical parameters from a sample shape is briefly described by Muinonen and Lagerros (1998). The mean radius $\tilde{a}$, the relative standard deviation of radius $\tilde{\sigma}$, and the standard deviation of slopes $\tilde{\rho}$ of an individual shape are given by the following relationships:

$$\tilde{a} = E(r)$$
$$\tilde{\sigma}^2 = \frac{1}{\tilde{a}^2}[E(r^2) - E(r)^2] \tag{3}$$
$$\tilde{\rho}^2 = \frac{1}{2}E\left(\frac{r_\theta^2}{r^2} + \frac{r_\varphi^2}{r^2\sin^2\theta}\right)$$

where $E((r(\theta, \varphi))$ is the intrinsic expectation of radius $r(\theta, \varphi)$

$$E(r) = \frac{1}{4\pi}\int_0^{\pi}\int_0^{2\pi} r(\theta,\varphi)\sin\theta\, d\theta\, d\varphi$$

r_θ and r_φ are the partial derivatives of the radius. The standard deviations of radius and slope can be related to the correlation angle $\tilde{\Gamma}$

Table 2
Best-fit dumb-bell shaped solutions from simulated lightcurves of near-equilibrium DB-shaped bodies for a nominal aspect angle $\psi = 85°$ and a scattering parameter $k = 0.24$. Statistical parameters of the Gaussian random sphere (σ_{GRS}, Φ_{GRS}, Γ_{GRS}) and the resulting DB-shaped bodies ($\tilde{\sigma}, \tilde{\Phi}, \tilde{\Gamma}$) are shown for two values of Ω and three standard deviations of radius σ_{GRS} of the associated GRS.

Ω	σ_{GRS}	Φ_{GRS} (°)	Γ_{GRS} (°)	$\tilde{\sigma}$	$\tilde{\Phi}$ (°)	$\tilde{\Gamma}$ (°)	Ω_{fit}	k_{fit}	ψ_{fit} (°)	Std. dev. (mag)	χ_{2r}
0.29	0.097	12.9	24.5	0.85	41.1	50.3	0.287 ± 0.002	0.17 ± 0.08	83.3 ± 2.2	0.021	1.1
	0.16	20.7	24.2	0.82	41.7	47.7	0.287 ± 0.002	0.24 ± 0.10	83.1 ± 2.2	0.039	3.8
	0.22	27.5	23.8	0.79	42.9	44.1	0.287 ± 0.002	0.23 ± 0.10	82.5 ± 2.2	0.045	4.6
0.36	0.12	17.0	23.4	0.43	34.3	34.9	0.348 ± 0.010	0.29 ± 0.08	82.9 ± 1.9	0.025	1.6
	0.18	24.9	21.8	0.37	33.5	31.2	0.346 ± 0.012	0.18 ± 0.07	78.9 ± 2.7	0.032	2.7
	0.24	32.7	21.5	0.35	36.1	27.2	0.311 ± 0.015	0.56 ± 0.20	78.8 ± 2.7	0.057	8.4