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Theory for planetary exospheres: II. Radiation pressure effect on exospheric density profiles

A. Beth [a,b,c,*], P. Garnier [a,b,1], D. Toublanc [a,b], I. Dandouras [a,b], C. Mazelle [a,b]

[a] Université de Toulouse; UPS-OMP; IRAP, Toulouse, France
[b] CNRS; IRAP, 9 Av. Colonel Roche, BP 44346, F-31028 Toulouse Cedex 4, France
[c] Department of Physics/SPAT, Imperial College London, United Kingdom

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ABSTRACT

The planetary exospheres are poorly known in their outer parts, since the neutral densities are low compared with the instruments detection capabilities. The exospheric models are thus often the main source of information at such high altitudes. We present a new way to take into account analytically the additional effect of the radiation pressure on planetary exospheres. In a series of papers, we present with an Hamiltonian approach the effect of the radiation pressure on dynamical trajectories, density profiles and escaping thermal flux. Our work is a generalization of the study by Bishop and Chamberlain (1989). In this second part of our work, we present here the density profiles of atomic Hydrogen in planetary exospheres subject to the radiation pressure. We first provide the altitude profiles of ballistic particles (the dominant exospheric population in most cases), which exhibit strong asymmetries that explain the known geotail phenomenon at Earth. The radiation pressure strongly enhances the densities compared with the pure gravity case (i.e. the Chamberlain profiles), in particular at noon and midnight. We finally show the existence of an exopause that appears naturally as the external limit for bounded particles, above which all particles are escaping.

1. Introduction

The exosphere is the upper layer of any planetary atmosphere: it is a quasi-collisionless medium where the particle trajectories are more dominated by gravity than by collisions. Above the exobase, the lower limit of the exosphere, the Knudsen number (Ferziger and Kaper, 1972) becomes large, collisions become scarce, the distribution function cannot be considered as Maxwellian any more and, gradually, the trajectories of particles are essentially determined by the gravitation and radiation pressure by the Sun. The trajectories of particles, subject to the gravitational force, are completely solved with the equations of motion, but it is not the case with the radiation pressure (Bishop and Chamberlain, 1989).

To describe correctly the exospheric population, we distinguish three types of particles: escaping, ballistic and satellite (Chamberlain, 1963; Banks and Kockarts, 1973).

- The escaping particles come from the exobase and have a positive mechanical energy: they can escape from the gravitational influence of the planet with a velocity larger than the escape velocity. These particles are responsible for the Jeans' escape (Jeans, 1916).
- The ballistic particles also come from the exobase but with a negative mechanical energy, they are gravitationally bound to the planet. They reach a maximum altitude and fall down on the exobase if they do not undergo collisions.
- The satellite particles never cross the exobase. They also have a negative mechanical energy but their periapsis is above the exobase: they orbit along an entire ellipse around the planet without crossing the exobase. The satellite particles result from ballistic particles undergoing few collisions mainly near the exobase. Thus, they do not exist in a collisionless model of the exosphere.

By definition, their trajectories are conics in the pure gravity case. Chamberlain (1963) proposed an analytical approach to estimate the density of each population via Liouville's theorem which states that the distribution function remains constant along a dynamical trajectory. A Maxwellian distribution function is assumed at the exobase and propagated to the upper layers via

* Principal corresponding author at: Department of Physics/SPAT, Imperial College London, United Kingdom.

E-mail addresses: arnaud.beth@gmail.com (A. Beth), pgarnier@irap.omp.eu (P. Garnier).

[1] Corresponding author at: Université de Toulouse; UPS-OMP; IRAP, Toulouse, France.

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Liouville's theorem. The density for each population is then derived as the product between the barometric law and a partition function ζ.

$$n(r) = n_{bar}\zeta(\lambda) = n(r_{exo})e^{\lambda-\lambda_{exo}}(\zeta_{bal} + \zeta_{esc}) \tag{1}$$

where λ is the ratio between the gravitational and thermal energies.

$$\lambda(r) = \frac{GMm}{k_B T_{exo} r} = \frac{v_{esc}(r)^2}{U^2} \tag{2}$$

with r the distance from the center of the body, $v_{esc}(r)$ the escaping velocity, U the most probable velocity for the Maxwellian distribution, G the gravitational constant, M the mass of the planet or the satellite and T_{exo} the temperature at the exobase considered constant in the exosphere.

The radiation pressure disturbs the ellipses or hyperbolas described by these particles. The resonant scattering of solar photons leads to a total momentum transfer from the photon to the atom or molecule. In the non-relativistic case, assuming an isotropic reemission of the solar photon, this one is absorbed in the Sun direction and scattered with the same probability in all directions. For a sufficient flux of photons in the absorption wavelength range, the reemission in average does not induce any momentum transfer from the atom/molecule to the photon. The differential of momentum between before and after the scattering each second imparts a force, the radiation pressure. Bishop and Chamberlain (1989) proposed to take into account this effect on the structure of planetary exospheres. In particular, they highlighted analytically the "tail" phenomenon: the density for atomic Hydrogen is higher in the nightside direction than in the dayside direction, as observed for the first time by OGO-5 (Thomas and Bohlin, 1972; Bertaux and Blamont, 1973).

This problem is similar to so-called Stark effect (Stark, 1914): the effect of a constant electric field on the atomic Hydrogen's electron. Its study shows it can be transposed to celestial mechanics in order to describe the orbits of artificial and natural satellites in the perturbed two-body problem. A first but incomplete work was performed by Bishop and Chamberlain (1989). They focused on the density profiles along the Sun-planet axis: in the velocity phase space, the problem is thus only 2D (one component of the angular momentum is null, p_ϕ, and thus the problem takes place on a hyperplane in the 3D-velocity phase space). They determined the density profiles for bounded trajectories (only ballistic particles, neither escaping nor satellite particles) for atomic Hydrogen along the Sun-planet axis, on the dayside and the nightside, for Earth, Venus, Mars or for sodium at Mercury.

In this work, we generalize the formalism developed by Bishop and Chamberlain (1989) to the whole exosphere (3D case) and highlight several phenomena. Our study is based on Beth et al. (2016), where we detailed the dynamical aspects induced by the radiation pressure on the trajectories of exospheric particles. We now present the implications on exospheric density profiles, local time asymmetries as well as a specific study of the particles with satellite orbits. We will briefly describe the formalism used in Section 2, before we derive the neutrals density in Section 3, and present the results in Section 4 and conclude in Section 5.

2. Model

For this work, we decide to study the effect of the radiation pressure on atomic Hydrogen in particular. We model the radiation pressure by a constant acceleration a coming from the Sun. As previously defined by Bishop (1991), this acceleration depends on the line center solar Lyman-α flux f_0, in 10^{11} photons cm^{-2} s^{-1} Å^{-1}:

$$a = 0.1774\, f_0\ (\mathrm{cm\ s^{-2}}) \tag{3}$$

This problem is similar to the classical Stark effect (Stark, 1914): a constant electric field (here the radiation pressure) applied on an electron (here an Hydrogen atom) attached to a proton (here the planet). Both systems are equivalent because the force applied by the proton (the planet) on the electron (the Hydrogen atom), the electrostatic force, varies in r^{-2} as the gravitational force from the planet on the Hydrogen atom. Thus, we adopt the same formalism as Sommerfeld (1934) adopting the parabolic coordinates. We use the transformation:

$$\begin{aligned} u &= r + x = r(1+\cos\theta) \\ w &= r - x = r(1-\cos\theta) \end{aligned} \tag{4}$$

where x is the sunward coordinate and θ the angle with respect to the Sun-planet axis. Along the Sun-planet axis, w is null in the sunward direction whereas u is null in the nightside direction. Consequently, the Hamiltonian becomes:

$$\mathcal{H}(u, w, p_u, p_w, p_\phi) = \frac{2up_u^2 + 2wp_w^2}{m(u+w)} + \frac{p_\phi^2}{2muw} - \frac{2GMm}{u+w} + ma\frac{u-w}{2} \tag{5}$$

independent of t, the time and ϕ, the azimuth about the planet-Sun axis. p_u, p_w and p_ϕ are the conjugate momenta, GM the standard gravitational parameter of the planet and m the mass of the species.

According to canonical relations, we have:

$$\begin{aligned} p_u &= \frac{m(u+w)}{4u}\frac{\mathrm{d}u}{\mathrm{d}t} \\ p_w &= \frac{m(u+w)}{4w}\frac{\mathrm{d}w}{\mathrm{d}t} \\ p_\phi &= muw\frac{\mathrm{d}\phi}{\mathrm{d}t} \end{aligned} \tag{6}$$

We do not assume $p_\phi = 0$ as Bishop and Chamberlain (1989) did: their study is restricted to the Sun-planet axis where either $u = 0$ or $w = 0$.

As shown by Bishop and Chamberlain (1989), the problem has three constants of the motion: E the mechanical energy, p_ϕ and A defined as

$$E = \mathcal{H} \tag{7}$$

because the forces are conservative,

$$\begin{aligned} A &= 2muE - 4up_u^2 - \frac{p_\phi^2}{u} - m^2au^2 + 2GMm^2 \\ &= -2mwE + 4wp_w^2 + \frac{p_\phi^2}{w} - m^2aw^2 - 2GMm^2 \end{aligned} \tag{8}$$

a separation constant (Bishop and Chamberlain, 1989) which is similar to the norm of the Laplace–Runge–Lenz vector and p_ϕ because

$$\frac{\mathrm{d}p_\phi}{\mathrm{d}t} = -\frac{\partial\mathcal{H}}{\partial\dot{\phi}} = 0 \tag{9}$$

As function of these three constants, we can rewrite the conjugate momenta:

$$\begin{aligned} p_u &= \pm\sqrt{\frac{-P_3(u)}{4u^2}} \\ p_w &= \pm\sqrt{\frac{Q_3(w)}{4w^2}} \end{aligned} \tag{10}$$

with

$$\begin{aligned} P_3(u) &= mau^3 - 2mEu^2 - (2GMm^2 - A)u + p_\phi^2 \\ Q_3(w) &= maw^3 + 2mEw^2 + (2GMm^2 + A)w - p_\phi^2 \end{aligned} \tag{11}$$