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Geography of the rotational resonances and their stability in the ellipsoidal full two body problem

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ABSTRACT

A fourth-order Hamiltonian describing the planar full two body problem is obtained, allowing for a mapping out of the geography of spin–spin–orbit resonances. The expansion of the mutual potential function up to the fourth-order results in the angles to come through one single harmonic and consequently the rotation of both bodies and mutual orbit are coupled. Having derived relative equilibria, stability analysis showed that the stability conditions are independent of physical and orbital characteristics. Simultaneously chaotic motion of bodies is investigated through the Chirikov diffusion utilizing geographic information of the complete resonances. The results show that simultaneous chaos among the binary asteroids is not expected to be prevalent due to the mass distribution of primary in compare with secondary. If mass distribution of bodies is of the same order, simultaneous chaos and global instability are achievable.

1. Introduction

Full Two Body (F2B) Problem is defined as the unfolding the dynamics of two unconstrained rigid bodies which move around each other based on gravitational interaction between them (Scheeres, 2009). This problem has applications in many fields; for example, the binary asteroids systems (Bellerose and Scheeres, 2008; Scheeres, 2002). Due to irregular shape and mass characteristics of the binary asteroids, the gravitational interaction in the binary system has distinct attraction from a nonlinear dynamics point of view and merit discussing. By modeling binary asteroids as an ellipsoid–ellipsoid model, the rotational dynamics can exhibit non-trivial behavior.

Thanks to the Maciejewski, Scheeres and some others, a number of studies of the F2B problem and binary asteroids have been made. In his seminal work, Maciejewski (1995) formulated the general equations of motion of a system that consists of a finite number of extended rigid bodies by using the relative coordinates to reduce the number of variables. He showed that the equations of motion of two rigid bodies have Hamiltonian structure and proved the existence of relative equilibria for reduced equations. The Maciejewski's work is the basis of most of work in the study of F2B problem and binary asteroids (Scheeres, 2002, 2004; Gabern et al., 2006). Scheeres (2002) derived stability and instability conditions in the Hill's sense for the F2B problem based on classical results from the N-body problem. Moreover, he investigated the stability against impact.

Bellerose and Scheeres (2008) computed families of periodic orbits and also derived the conditions for relative equilibria and their stability in the F2B system as a function of the mass ratio. They considered ellipsoid–sphere model and the mutual potential in terms of elliptic integrals. Fahnestock and Scheeres (2008) presented a method for numerical simulation of the fully coupled rotational and orbital dynamics of 1999-KW4 binary asteroid systems based on the polyhedral mutual potential (Werner and Scheeres, 2005). They also developed analytical formulation for studying the effects of primary's oblateness and secondary's triaxiality, independently.

Boue and Laskar (2009) studied long term evolution of spin axis of two interacting bodies through the ellipsoids–ellipsoids model. They utilized a potential function which is expanded up to the fourth-order to find the contribution of the relative

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orientation of the bodies in the dynamics of the system. They derived the secular evolutionary equation of the spin axis of both bodies using the averaging method. They showed that the secular evolution is integrable and composed of a global precession of the three angular momentum and nutation loops which are periodic. They used a specific range of the triaxiality and the semi-major axis to observe regular solution. Scheeres (2009) studied relative equilibria and their stability in a planar F2B problem which in the bodies are non-spherical. The mutual potential function is expanded up to the second-order and the energetically stable relative equilibria and the conditions for the Hill stability of the system are found.

Recently Batygin and Morbidelli (2015) investigated the rotational evolution of binary system in the Solar System and introduced spin–spin resonances through the quadrupole–quadrupole interactions. They considered a planar system including an aspherical secondary moving around a rotating aspherical primary in a circular orbit. Due to negligible mass assumption of the secondary, only its spin evolves under the influence of the primary's gravitational potential. They also derived the spin–spin resonance capture probabilities by considering the constant time-lag tidal model.

The tidal effects (Batygin and Morbidelli, 2015; Taylor and Margot, 2014), YORP, and binary YORP (Steinberg and Sari, 2011) have an important role in the evolution of binary systems. However, for the sake of simplicity, this paper focuses on the purely conservative system.

There are several studies with valuable results in the field of F2B problem and binary asteroids. Nonetheless, no research can be found that addresses the resonant dynamics and its stability by considering the fourth-order of gravitational potential in which the coupling of rotation of both bodies and mutual orbit is evident. This paper deals with the investigation of spin–spin–orbit (SSO) resonance within planar F2B system. In this study, the SSO resonance is considered similar to those of spin–orbit problem (Goldreich and Peale, 1966; Wisdom and Peale, 1984; Naidu and Margot, 2015). Two bodies move on a specified Keplerian orbit around their center of mass whereas the rotation of them is supposed to be around its maximum moment of inertia axis, perpendicular to the orbital plane. Therefore, the system under study is planar.

Even though the dynamical behavior is influenced by the asphericity and orbital eccentricity in the spin–orbit problem (Jafari Nadoushan and Assadian, 2015), the asphericities of both bodies, ε_1 and ε_2, the semi-major axis and the eccentricity of mutual orbit are the factors which play key role in the dynamics of F2B problem. In this paper, the effects of these parameters on the existence and stability of various resonances and diffusion phenomena in the F2B system are investigated. It should be noted that this paper is specifically concerned with Chirikov diffusion arising exclusively from overlap of SSO resonances. In short, the main contribution of this study is the articulation of the potential function in which the resonances appear, and studying the appearance of chaos only in rotational dynamics coupled with orbital motion (SSO resonances).

This paper is organized as follows; in Section 2, the classical Hamiltonian formalism is used for deriving the rotational dynamics of the F2B system. The resonances are isolated and their nonlinear stability is discussed in Section 3. In Section 4, the strength of the resonances is derived followed by the discussion on occurrence of the Chirikov diffusion and simultaneous chaos in the F2B problem in Section 5. Section 6gives the conclusion of this paper.

2. Spin–spin–orbit dynamics

The geometry of a planar motion of two ellipsoidal bodies around the center of their mass for prograde orbit is depicted in Fig. 1. Both bodies are symmetric about their equatorial plane and their moments of inertia are $A_i < B_i < C_i$ where $i = 1, 2$. Such system requires two coordinates to describe the motion of the mass center and two for the rotational motion. It is assumed that the bodies are moving in a specified fixed orbit with semi-major axis a and eccentricity e. Furthermore, the rotation of them are supposed to be about their maximum moment of inertia axis, perpendicular to the orbital plane.

2.1. Potential energy

The mutual potential of two general rigid bodies β_1 and β_2 with the mass m_1 and m_2, respectively, takes the form:

$$U = -\int_{\beta_1}\int_{\beta_2}\frac{G}{\Delta}dm_2dm_1 \tag{1}$$

where G is the universal gravitational constant and $\Delta = \left|\vec{\Delta}\right| = \left|\vec{r} - \left(\vec{\rho}_1 - \vec{\rho}_2\right)\right|$ in which $\vec{r}$ is the instantaneous radius vector from the center of mass of the β_1 to the center of mass of the β_2, and $\vec{\rho}_i$ is the relative position vector of dm_i with respect to the center of mass of the β_i (see Fig. 2). This integral can be calculated using the binomial theorem to expand the denominator of the potential function in the Legendre polynomials. The detailed derivation of mutual potential energy of the ellipsoidal F2B system up to the fourth-order is presented in the appendix. However, the potential energy of the planar system is used in this section based on the appendix formulations. It is noteworthy to mention that since this paper concerns only about the rotational motion of bodies, those terms of potential energy are considered which contribute on the rotational motion.

For this purpose, the origin of reference frame is considered to be coincided with the center of mass of the system and the x axis of the reference frame is taken as the orbital radius vector, thus the unit vector connecting the two bodies is defined as $\hat{r} = [1, 0, 0]^T$ and the transformation matrixes from the reference frame to the body frame β_i is defined by a single rotation angle ψ_i (Fig. 1), as:

$$T_{ref}^{\beta_i} = \begin{bmatrix} \cos(\pi-\psi_i) & \sin(\pi-\psi_i) & 0 \\ -\sin(\pi-\psi_i) & \cos(\pi-\psi_i) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

The transformation matrixes from the body frame β_2 to the body frame β_1 is

$$T_{\beta_2}^{\beta_1} = \begin{bmatrix} \cos(\psi_1-\psi_2) & -\sin(\psi_1-\psi_2) & 0 \\ \sin(\psi_1-\psi_2) & \cos(\psi_1-\psi_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

After substituting Eqs. (2) and (3) into Eqs. (47), (51)–(53) and summing them, and considering those terms which contribute on the rotational motion, the potential energy up to the fourth-order for the planar system is found as