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GRAIL gravity field determination using the Celestial Mechanics Approach

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ABSTRACT

The NASA mission GRAIL (Gravity Recovery and Interior Laboratory) inherited its concept from the GRACE (Gravity Recovery and Climate Experiment) mission to determine the gravity field of the Moon.

We present lunar gravity fields based on the data of GRAIL's primary mission phase. Gravity field recovery is realized in the framework of the Celestial Mechanics Approach, using a development version of the Bernese GNSS Software along with Ka-band range-rate data series as observations and the GNI1B positions provided by NASA JPL as pseudo-observations.

By comparing our results with the official level-2 GRAIL gravity field models we show that the lunar gravity field can be recovered with a high quality by adapting the Celestial Mechanics Approach, even when using pre-GRAIL gravity field models as a priori fields and when replacing sophisticated models of non-gravitational accelerations by appropriately spaced pseudo-stochastic pulses (i.e., instantaneous velocity changes).

We present and evaluate two lunar gravity field solutions up to degree and order 200 – AIUB-GRL200A and AIUB-GRL200B. While the first solution uses no gravity field information beyond degree 200, the second is obtained by using the official GRAIL field GRGM900C up to degree and order 660 as a priori information. This reduces the omission errors and demonstrates the potential quality of our solution if we resolved the gravity field to higher degree.

1. Introduction

The NASA mission Gravity Recovery and Interior Laboratory (GRAIL) is the latest and most sophisticated satellite mission for the determination of the lunar gravity field (Zuber et al., 2013b). It consists of two satellites, GRAIL-A ("Ebb") and GRAIL-B ("Flow"), launched on 10 September 2011, and entering nearly polar lunar orbits in the beginning of 2012. Data for gravity field recovery was acquired during two science phases, given by the requirement that the elevation of the Sun above the orbital plane is large enough for the solar panels to produce enough power. In the primary mission (PM) phase (1 March to 29 May 2012) the mean altitude of the two satellites was 55 km above the lunar surface (corresponding to a revolution period of about 113 min). The eccentricity of the orbits varied between around 0.02 at the beginning and end of the PM phase and 0.003 in the middle, while the inclination changed between 89.9° and 88.5° with a bimonthly period. In the extended mission (EM) phase (30 August to 14 December 2012) the orbits were lowered to 23 km on average (Zuber et al., 2013a). The separation between the two spacecraft varied from 40 km to 220 km during the mission to find a compromise between sensitivity and multipath effects on the inter-satellite communications (Konopliv et al., 2013). The mission ended with a controlled crash of the two probes on the Moon on 17 December 2012.

The concept of the mission was inherited from the Earth-orbiting mission Gravity Recovery and Climate Experiment (GRACE, Tapley et al., 2004) in that the key observations consisted of ultra-precise inter-satellite Ka-band range measurements (Asmar et al., 2013). The use of these observations enables data acquisition even when the spacecraft are not Doppler-tracked from the Earth. Together with the one- and two-way Doppler observations from the NASA Deep Space Network (DSN), GRAIL data allows for a determination of the lunar gravity field with an unprecedented accuracy for both the near- and the far-side of the Moon. Its results are essential to improve the understanding of the Moon's internal structure and thermal evolution (Wieczorek et al., 2013).

The official GRAIL gravity field models based on PM data consist of spherical harmonic (SH) coefficients up to degree and order 660 [GL0660B (Konopliv et al., 2013) and GRGM660PRIM (Lemoine

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et al., 2013)], while the latest models resolve the selenopotential up to degree and order 900 [GL0900D (Konopliv et al., 2014) and GRGM900C (Lemoine et al., 2014)] by using both PM and EM data. These solutions were obtained using the software packages MIRAGE, a gravity processing version of the JPL Orbit Determination Program (ODP, Moyer, 2003), and GEODYN (Pavlis et al., 2013), respectively.

Apart from the Ka-band range and Doppler data, satellite positions (GNI1B positions) of the GRAIL probes from the dynamic orbit determination performed at JPL are also available. As explained by Jäggi et al. (2015), the usage of these positions as pseudo-observations allows for a relatively straightforward adaption of our gravity field recovery procedures from GRACE (Jäggi et al., 2008), where GPS-derived kinematic positions are available, to GRAIL without having first to implement Doppler data processing. These adaptions are performed within a development version of the Bernese GNSS Software (Dach et al., 2007). The use of GNI1B positions is only considered an intermediate solution for our approach as it will not lead to a completely independent gravity field solution.

In addition to normalized spherical harmonic coefficients up to degree 200, we set up arc- and satellite-specific parameters (like initial state vectors and pseudo-stochastic pulses) as common parameters for all measurement types (see Section 2 for more details). Pseudo-stochastic pulses shall compensate for imperfect models of non-gravitational accelerations, e.g., caused by solar radiation pressure.

This paper is structured as follows. In Section 2 we briefly review the Celestial Mechanics Approach (CMA) for gravity field determination. Section 3 discusses some results for the initial orbit determination of the GRAIL satellites and outlines the performance of our data modeling, paying attention to possible limitations due to missing radiation pressure modeling. We point out that up to release 2 of the level-1b data the estimation of a Ka-band time bias is mandatory (see Section 3.3). In Section 4 we focus on gravity fields obtained by using the CMA. We present our lunar gravity field solutions up to degree and order 200. Section 5 gives an outlook on possible further improvements to our orbit and gravity field modeling. Section 6 presents our final remarks and conclusions.

2. The Celestial Mechanics Approach

The Celestial Mechanics Approach (CMA, Beutler et al., 2010) treats gravity field recovery as an extended orbit determination problem. It is a dynamic approach allowing for pseudo-stochastic parameters to absorb force model deficiencies.

For a GRAIL probe the equations of motion in the inertial system read as

$$\ddot{\vec{r}} = -GM_M \frac{\vec{r}}{r^3} + \vec{f}(t, \vec{r}, \dot{\vec{r}}, q_1, \ldots, q_d), \tag{1}$$

where GM_M denotes the gravity parameter of the Moon, $\vec{r}$ is the selenocentric position of the probe and $\vec{f}$ collects all perturbing accelerations as described in Section 2.1. Dots indicate derivatives w.r.t. time. The second-order differential Eq. (1) require six initial or boundary conditions for a particular solution. In the framework of the CMA satellite motion is described as an initial value problem.[1] The initial conditions $\vec{r}(t_0) = \vec{r}(a, e, i, \Omega, \omega, u; t_0)$ and $\dot{\vec{r}}(t_0) = \dot{\vec{r}}(a, e, i, \Omega, \omega, u; t_0)$ at an initial epoch t_0 are defined by six Keplerian osculating elements. a denotes the semi-major axis, e the numerical eccentricity, i the inclination w.r.t. the equatorial plane, Ω the right ascension of the ascending node, ω the argument of periapsis, and u the argument of latitude at time t_0. $q_1, \ldots, q_d$ parametrize the perturbing accelerations and are both arc-specific orbit parameters (e.g., empirical accelerations) and general parameters such as gravity field coefficients. Let us denote the $6+d$ parameters (initial conditions and q_i) collectively as p_i.

2.1. Perturbing accelerations

The perturbing accelerations $\vec{f}$ on the right-hand side of the equations of motion (1) are given by

$$\vec{f} = T_f^i \nabla V + \vec{a}_b + \vec{a}_t + \vec{a}_r + \vec{a}_n + \vec{a}_e, \tag{2}$$

where V denotes the lunar gravity potential in the Moon-centered body-fixed reference frame, T_f^i is a rotation matrix relating the Moon-centered body-fixed with the inertial system, $\vec{a}_b$ are the third-body accelerations, $\vec{a}_t$ denote accelerations due to the tidal deformation of the Moon, $\vec{a}_r$ are relativistic corrections, $\vec{a}_n$ non-gravitational accelerations and $\vec{a}_e$ empirical accelerations.

2.1.1. The gravity potential

The gravitational acceleration $\vec{a}_g \doteq T_f^i \nabla V$ exerted by the Moon is written in terms of the lunar gravity potential V, which is expressed using a standard spherical harmonics (SH) expansion (Heiskanen and Moritz, 1967) as

$$V(r, \lambda, \phi) = \frac{GM_M}{r} \sum_{l=1}^{l_{max}} \left(\frac{R_M}{r}\right)^l \sum_{m=0}^{l} \overline{P}_{lm}(\sin\phi) \cdot \left[\overline{C}_{lm}\cos(m\lambda) + \overline{S}_{lm}\sin(m\lambda)\right], \tag{3}$$

where R_M is the reference radius (1738 km) of the Moon and r, λ and ϕ denote the selenocentric radius, longitude and latitude of the evaluation point, respectively. $\overline{P}_{lm}$ are the fully normalized associated Legendre functions of degree l and order m, and $\overline{C}_{lm}$ and $\overline{S}_{lm}$ are the corresponding SH coefficients. For the lunar body-fixed reference system, we use the Principal Axes (PA) system with axes coinciding with the principal axes of inertia of the Moon. The Euler (libration) angles connecting the International Celestial Reference System (ICRS)[2] with the PA system are taken from the DE421 JPL ephemerides (Folkner et al., 2009).

In this article we present lunar gravity field solutions up to a maximum degree $l_{max} = 200$.

2.1.2. Third-body accelerations

We take third-body accelerations due to Earth, Sun, Jupiter, Venus and Mars into account. These celestial bodies are all approximated as point masses with positions taken from the JPL ephemerides DE421.

2.1.3. Tidal acceleration

The tidal deformation of the Moon due to other celestial bodies causes an additional acceleration acting on the GRAIL probes. According to the IERS2010 conventions (Petit and Luzum, 2010) this acceleration can be expressed in terms of a change of the (tide-free) SH gravity field coefficients as

$$\Delta\overline{C}_{lm} - i\Delta\overline{S}_{lm} = \frac{k_{lm}}{2l+1} \sum_{j=2}^{3} \frac{GM_j}{GM_M} \left(\frac{R_M}{r_j}\right)^{l+1} \cdot \overline{P}_{lm}(\sin\Phi_j) e^{-im\lambda_j}, \tag{4}$$

where j labels the perturbing body (Earth and Sun) of mass M_j. r_j, λ_j and Φ_j denote the spherical coordinates of the perturbing body in the lunar body-fixed system and the constants k_{lm} are the Love

[1] See Klinger et al. (2014) for a boundary value approach to GRAIL gravity field recovery.

[2] Aligned with the mean equator and dynamical equinox of J2000.0.