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Vertical structures induced by embedded moonlets in Saturn's rings

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ABSTRACT

We study the vertical extent of propeller structures in Saturn's rings (i) by extending the model of Spahn and Sremčević (Spahn, F., Sremčević, M. [2000]. Astron. Astrophys., 358, 368–372) to include the vertical direction and (ii) by performing N-body box simulations of a perturbing moonlet embedded into the rings. We find that the gravitational interaction of ring particles with a non-inclined moonlet does not induce considerable vertical excursions of ring particles, but causes a considerable thermal motion in the ring plane. We expect ring particle collisions to partly convert the lateral induced thermal motion into vertical excursions of ring particles in the course of a quasi-thermalization. The N-body box simulations lead to maximal propeller heights of about 0.6–0.8 Hill radii of the embedded perturbing moonlet. Moonlet sizes estimated by this relation are in good agreement with size estimates from radial propeller scalings for the propellers Blériot and Earhart. For large propellers, the extended hydrodynamical propeller model predicts an exponential propeller height relaxation, confirmed by N-body box simulations of non-self gravitating ring particles. Exponential cooling constants, calculated from the hydrodynamical propeller model agree fairly well with values from fits to the tail of the azimuthal height decay of the N-body box simulations. From exponential cooling constants, determined from shadows cast by the propeller Earhart and imaged by the Cassini spacecraft, we estimate collision frequencies of about 6 collisions per particle per orbit in the propeller gap region and about 11 collisions per particle per orbit in the propeller wake region.

1. Introduction

Planetary rings are one of the most remarkable and beautiful cosmic structures. They are natural dynamical laboratories (Burns and Cuzzi, 2006), exemplifying the physics of cosmic disks, such as accretion or galactic disks, which are much larger and much farther away from Earth. An exciting example is the presence of small moons embedded in Saturn's rings, henceforth called moonlets, which have their analog in planetary embryos orbiting within a protoplanetary disk (Artymowicz, 2006; Papaloizou, 2007).

The fact that spacecraft cameras (even Cassini's) do not have sufficient resolution to image these moonlets directly, brought up the idea of investigating moonlet-induced putative structures in the rings (Lissauer et al., 1981; Henon, 1981; Spahn, 1987; Petit and Henon, 1988; Spahn and Wiebicke, 1989), with the hope that these features could be captured by the spacecraft cameras or instruments. This then led to predictions of 'propeller'-shaped structures (Spahn and Sremčević, 2000; Sremčević et al., 2002) which are carved in the rings by the moonlet. Subsequent numerical particle experiments (Seiß et al., 2005; Sremčević et al., 2007; Lewis and Stewart, 2009) clarified the fingerprint of such gravitational perturbers and confirmed the spatial scaling of the propeller structure. Depending on its size, an embedded ring-moon either induces a partial gap (sizes below a few km) or, alternatively, opens up a complete circumferential gap (for sizes above a few km, e.g. the ring-moons *Pan* and *Daphnis*). Both structures are decorated with density wakes, completing the structural picture. Up to this stage, analytical study of propellers has focused exclusively on structure within the ring plane.

More than 150 propellers have now been detected (Tiscareno et al., 2006; Tiscareno et al., 2008; Sremčević et al., 2007) and among them a few which are large enough to be seen on several snapshots taken by *Cassini's* cameras at different times, confirming in this way their orbital motion. Those propellers were nicknamed after famous aviators, e.g.: Blériot, Kingsford Smith, Earhart (Tiscareno et al., 2010).

In the summer of 2009, at Saturn's equinox (the sunset at Saturn's rings), the perfect opportunity arose to detect any vertical

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structure deviating from the mean ring plane by observing shadows cast on the rings. At this time the density structures around the largest propeller moonlets, as well as those around the ringmoon *Daphnis*, created prominent shadows. These can be assigned to the wakes and in the case of the propeller moonlets also to excited regions of the propeller, where the moonlet induces two partial gaps. The shadows were much longer than the moon's size itself, leading to the conclusion that moonlet-induced vertical excursions of ring particles can be in the range of several kilometers in the case of *Daphnis* or several hundred meters in the case of the large propeller moonlets. These very facts directly indicate the necessity to investigate the vertical stratification of moonlet-induced structures, which has not been the focus of former models of the moonlet's *fingerprint*.

Our study of the vertical extend of moonlet-induced propeller structures is organized as follows: In Section 2 the extended hydrodynamical propeller model is presented. In Section 2.2 we calculate the mass flow through the scattering region by a probabilistic approach and determine values of the moonlet-induced thermal velocities, which we use later as initial conditions for the hydrodynamical equations describing the long term relaxation of the moonlet-excited structures. Section 2.3 gives the hydrodynamical balance equations, which we use to model the diffusion of mass into the induced gap and the relaxation of the granular ring temperature. In Section 3 the azimuthal relaxation of the propeller height is calculated. Section 4 describes N-body box simulations of propeller moonlets embedded into the rings. These are used to verify assumptions made in the derivation of the extended propeller model and to compare results. Finally, we present and discuss the application of our results to observed propeller features in Saturn's rings in Section 5.

2. Extended model of gravitational scattering

2.1. The scattering region

The first step in the formulation of our model is to divide the planetary ring, composed of the ring particles and one moonlet, into two regions:

(i) the rather small scattering region,
(ii) the rest of the ring.

In this work we consider moonlets on circular and planar orbits, i.e. with zero eccentricity and zero inclination. The scattering region is the small area (volume) around the moonlet where trajectory changes due to the moonlet's gravity predominantly take place. This region of the embedded moonlet's gravitational influence is of the order of a few times the Hill radius

$$h = a_0\left(\frac{m_m}{3(m_m+m_s)}\right)^{1/3}, \tag{1}$$

where a_0 is the semimajor axis of the moonlet, m_m its mass and m_s the mass of Saturn. Compared to the moonlet's semimajor axis the Hill radius is usually very small. For large propellers, like Blériot or Earhart, the ratio $h^* = h/a_0$ is approximately 10^{-6} (Tiscareno et al., 2010). This low ratio naturally allows the splitting of the rings into the two regions, and further, it allows to regard the azimuthal extent of the scattering region to be zero, i.e. the approximation of the scattering region by a scattering line (Spahn and Wiebicke, 1989).

For the rest of the ring, where the moonlet's gravity is negligible, the moonlet-induced structures are assumed to relax due to inelastic collisional cooling and viscous diffusion (Spahn and Sremčević, 2000; Sremčević et al., 2002). The ring is regarded as a fluid and described with hydrodynamical balance equations, where the granular *temperature*[1] $T = c^2/3$ is used to describe the energy balance of the ring (Schmidt et al., 2009) and c denotes the velocity dispersion of the ring particles.

2.2. Encounter with the moonlet – gravitational scattering

We describe the encounter of ring particles with the moonlet in a corotating frame, rotating about Saturn with the Keplerian frequency $\Omega_0 = (Gm_s/a_0^3)^{1/2}$ of the moonlet. The dynamics of ring particles in the corotating frame is given by the equation

$$\ddot{\mathbf{r}} + 2\mathbf{\Omega}_0 \times \dot{\mathbf{r}} + \mathbf{\Omega}_0 \times (\mathbf{\Omega}_0 \times \mathbf{r}) = -\nabla\Phi_s - \nabla\Phi_m, \tag{2}$$

where $\mathbf{r}$ is the position vector of the ring particle relative to Saturn's center, $\mathbf{\Omega}_0$ is aligned with the planetocentric angular momentum of the moonlet and has magnitude Ω_0, and Φ_s and Φ_m are the gravitational potentials due to Saturn and the moonlet (assumed to be point masses, $\Phi_s = -Gm_s/r$, $\Phi_m = -Gm_m/s$, where s denotes the distance of the ring particle to the moonlet).

Hereby, for simplicity, we neglect Saturn's oblateness, which would result in slightly different mean motion, epicyclic frequency and vertical frequency (less than one percent difference for the semimajor axis of Earhart, respectively). It would also result in a moving pericenter and ascending node of the ring particle orbits, effects, which would be averaged-out in the calculation of the scattering operator in Section 2.2.2.

The ring particles are on orbits with low eccentricity and inclination and the mass of the moonlet is very small compared to Saturn's mass $m_m/m_s \ll 1$. Because we are interested in the ring particle motion in the vicinity of the moonlet, we fix the origin of the corotating frame to the mean orbital location of the moonlet. In the vicinity of the moonlet the equations of motion of ring particles are well approximated by Hill's equations (Hill, 1878; Hénon and Petit, 1986).

We assume that the x axis points radially outward, the y axis points into the azimuthal direction in which the moonlet is moving and the z axis is normal to the ring plane in such a way that the axes form a right-handed coordinate system. With the scaled coordinates $\tilde{x} = x/h, \tilde{y} = y/h, \tilde{z} = z/h$ and scaled time $t' = \Omega_0 t$, Hill's equations then read

$$\begin{aligned}
\ddot{\tilde{x}} &= 2\dot{\tilde{y}} + 3\tilde{x} - 3\tilde{x}/\tilde{s}^3 \\
\ddot{\tilde{y}} &= -2\dot{\tilde{x}} - 3\tilde{y}/\tilde{s}^3 \\
\ddot{\tilde{z}} &= -\tilde{z} - 3\tilde{z}/\tilde{s}^3,
\end{aligned} \tag{3}$$

where $\tilde{s}^2 = \tilde{x}^2 + \tilde{y}^2 + \tilde{z}^2$ is the scaled distance of the ring particle to the moonlet and $\dot{\tilde{x}} = d\tilde{x}/dt'$. These equations are point symmetric about the position of the moonlet ($\tilde{x} = \tilde{y} = \tilde{z} = 0$), and quite comfortably, they do not depend on the moonlet mass anymore. All information of the moonlet mass is contained in the scaling length h.

When the ring particles are not in the vicinity of the moonlet, i.e. $1/\tilde{s}^3 \to 0$ and therefore $|\nabla\Phi_m| \to 0$, their trajectories are well described by the solutions to the homogeneous Hill's equations

$$\begin{aligned}
\tilde{x}(t') &= \tilde{a} - \tilde{e}\cos(t' + \psi) \\
\tilde{y}(t') &= C - \frac{3}{2}\tilde{a}t' + 2\tilde{e}\sin(t' + \psi) \\
\tilde{z}(t') &= \tilde{\imath}\sin(t' + \zeta).
\end{aligned} \tag{4}$$

The semimajor axis, eccentricity and inclination are scaled according to

[1] The granular temperature is a measure of the random motion of the (macroscopic) ring particles and should not be confused with the thermodynamic temperature.