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Insolation patterns on eccentric exoplanets

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ABSTRACT

Several studies have found that synchronously-rotating Earth-like planets in the habitable zones of M-dwarf stars should exhibit an "eyeball" climate pattern, with a pupil of open ocean facing the parent star, and ice everywhere else. Recent work on eccentric exoplanets by Wang et al. (Wang, Y., Tian, F., Hu, Y. [2014b] Astrophys. J. 791, L12) has extended this conclusion to the 2:1 spin–orbit resonance as well, where the planet rotates twice during one orbital period. However, Wang et al. also found that the 3:2 and 5:2 half-odd resonances produce a zonally-striped climate pattern with polar icecaps instead. Unfortunately, they used incorrect insolation functions for the 3:2 and 5:2 resonances whose long-term time averages are essentially independent of longitude.

This paper presents the correct insolation patterns for eccentric exoplanets with negligible obliquities in the 0:1, 1:2, 1:1, 3:2, 2:1, 5:2, 3:1, 7:2, and 4:1 spin–orbit resonances. I confirm that the mean insolation is distributed in an eyeball pattern for integer resonances; but for half-odd resonances, the mean insolation takes a "double-eyeball" pattern, identical over the "eastern" and "western" hemispheres. Presuming that liquids, ices, clouds, albedo, and thermal emission are similarly distributed, this has significant implications for the observation and interpretation of potentially habitable exoplanets.

Finally, whether a striped ball, eyeball, or double-eyeball pattern emerges, the possibility exists that long-term build-up of ice (or liquid) away from the hot spots may alter the planet's inertia tensor and quadrupole moments enough to re-orient the planet, ultimately changing the distribution of liquid and ice.

1. Introduction

Recent work suggests that planets at ~0.1 Astronomical Unit from M-type stars may be habitable (Dressing and Charbonneau, 2013; Gaidos, 2013; Kopparapu, 2013; Tuomi et al., 2014, etc.). So close to a star, however, gravitational tides are expected to lock a solid planet into a spin–orbit resonance state such that either end of its principal axis of least inertia (or its intermediate axis of inertia in certain cases) always points toward its sun at perihelion.

Then the planet's orbital period P is an integer or half-odd-integer multiple p of its rotation period; thus $p = \omega/n$, where ω is the planet's rotation rate and $n = 2\pi/P$ is its "mean motion", or average orbital angular velocity. For example, our Moon rotates once every 27 days, the same period as its orbit, so that it always keeps the same face toward Earth; in fact, most of the major moons in the Solar System are in the same "synchronous" $p = 1$ resonance. In contrast, the planet Mercury is in the "sesqui-synchronous" $p = 3/2$ resonance, such that it rotates three times during every two orbits.

The probability of finding a moon or planet in any particular resonance depends on its orbital eccentricity e (e.g., Dobrovolskis, 1995, 2007). For example, the synchronous $p = 1$ resonance is the only end state possible for planets in circular orbits, where e vanishes. For a planet in this synchronous state, the hemisphere facing the star would be in permanent sunlight, while the opposite hemisphere would be in eternal night. Climate models suggest that an Earth-like planet in this situation would experience an "eyeball" climate pattern, with a pupil of open ocean facing the parent star, and ice everywhere else (Pierrehumbert, 2011; Edson et al., 2011, 2012; Heng and Vogt, 2011; Hu and Yang, 2014, etc.).

However, many extra-solar planets have quite eccentric orbits ($e \gtrsim 0.2$). For such exoplanets, the sesqui-synchronous $p = 3/2$ state or a higher-order resonance is more likely than the synchronous $p = 1$ state. It is not obvious what sort of climate to expect for these non-synchronous spin–orbit resonances.

Recently, Wang et al. (2014a,b) have used a General Circulation Model to simulate the climate on planets orbiting an M-dwarf star with eccentricity $e = 0.40$, but zero obliquity, in the $p = 1$, 3/2, 2, and 5/2 spin–orbit resonances. They found that the $p = 1$ and

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$p = 2$ resonances both produce an eyeball climate pattern, but that the $p = 3/2$ and $p = 5/2$ resonances both produce a zonally-striped climate pattern with polar icecaps instead. Unfortunately, they used incorrect insolation functions for the 3/2 and 5/2 resonances whose long-term time averages are essentially independent of longitude.

This paper presents the correct insolation patterns for exoplanets again with $e = 0.40$, but negligible obliquities, in the $p = 0$, 1/2, 1, 3/2, 2, 5/2, 3, 7/2, and 4 spin–orbit resonances. I confirm that the mean insolation is distributed in an eyeball pattern for integer resonances; but for half-odd resonances, the mean insolation takes a "double-eyeball" pattern, identical over the "eastern" and "western" hemispheres. For such planets, the climate may assume a similar double-eyeball pattern.

2. Sub-solar point

By Kepler's second law (equivalent to conservation of angular momentum), a planet travels around its parent star (or equivalently, its sun travels around the planet) at an angular velocity

$$dv/dt = \ell/r^2, \tag{1}$$

where the constant

$$\ell = na^2\sqrt{1-e^2} \tag{2}$$

is the specific orbital angular momentum of the planet, and

$$r = \frac{a - ae^2}{1 + e\cos v} \tag{3}$$

is the varying distance between the star and planet.

Here $n = 2\pi/P$ is the planet's mean motion (time-averaged angular velocity), P is its orbit period (28 days in the case of GJ 667Cc used by Wang et al., 2014a,b), a is the semi-major axis of its orbit, e is its orbital eccentricity, and v is the planet's true anomaly: its longitude along its orbit, measured forward from perihelion. From the above relations, one can find the elapsed time t since perihelion explicitly as a function of v, or find v implicitly as a function of t.

Panel a of Fig. 1 plots the resulting time variation of the orbital angular velocity dv/dt for two whole orbits. The ordinate (left scale) gives $\dot{v} \equiv dv/dt$ in units of n, while the abscissa shows the elapsed time t since perihelion, in units of P (bottom scale), or as the mean anomaly nt in degrees of arc (top scale). Note that $\dot{v}/n$ ranges from just over 5/2 $\left(\sqrt{(1+e)/(1-e)^3} \approx 2.5459\right)$ to just under 1/2 $\left(\sqrt{(1-e)/(1+e)^3} \approx 0.4676\right)$ and back again during each orbit.

As seen from the surface of a planet rotating with angular velocity ω, the sub-solar point lies at a longitude $\phi_* = v - \omega t$ along the equator. In a spin–orbit resonance, where the sub-solar point lies on the axis of least (or intermediate) inertia at each perihelion, ϕ_* must be a strictly periodic function of time, with a period of one or two orbits. Therefore ω must be an integer or half-odd-integer multiple of the mean motion n.

Panels b–j of Fig. 1 plot ϕ_* versus time for the spin orbit resonances with $p \equiv \omega/n$ = 0, 1/2, 1, 3/2, 2, 5/2, 3, 7/2, and 4, respectively; note that my origin of longitude differs by ±180° from that of Wang et al. (2014a,b), so that the planet is at perihelion whenever $\phi_* = 0$. Panel b for the null resonance $p = 0$ (the antiderivative of panel a) shows that ϕ_* increases monotonically by 360° each orbit, except when it wraps around the "date line" at longitudes of ±180°.

Panel c for the "semi-synchronous" resonance $p = 1/2$ shows that ϕ_* generally increases by 180° per orbit, but lingers long near

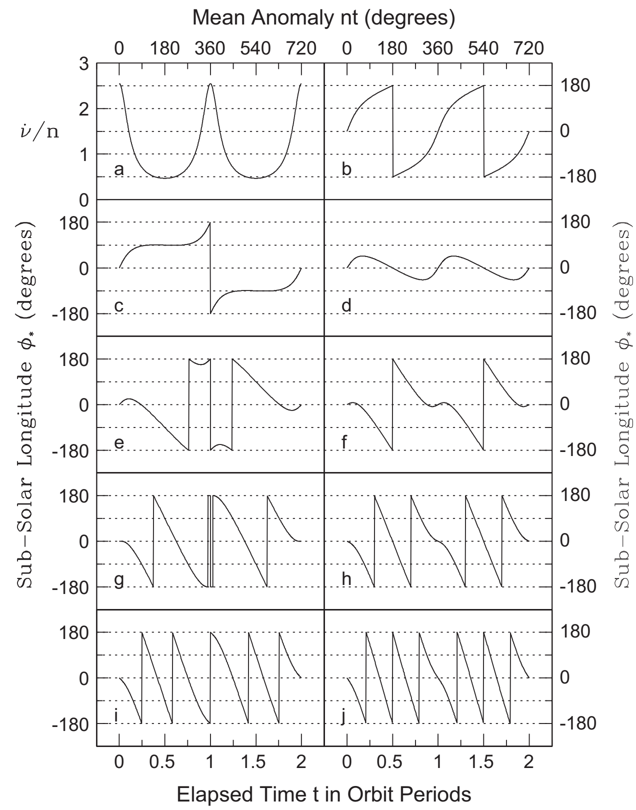

Fig. 1. Panel a: Inertial angular speed dv/dt of the planet around its sun, or of its sun around the planet, versus elapsed time t since perihelion. Panels b–j: Longitude of the sub-solar point on the planet's equator versus time for various resonant values of the planet's rotation rate $\omega = pn$, where $n = 2\pi/P$ is the planet's mean motion and P is its orbital period. Panel b: $p = 0$. Panel c: $p = 1/2$. Panel d: $p = 1$. Panel e: $p = 3/2$. Panel f: $p = 2$. Panel g: $p = 5/2$. Panel h: $p = 3$. Panel i: $p = 7/2$. Panel j: $p = 4$.

+90° on one orbit, and −90° on the next; in fact, ϕ_* makes small retrograde loops with an amplitude of ~0°.84 during the intervals when $dv/dt < n/2$. For comparison, panel d for the synchronous state $p = 1$ shows that ϕ_* librates about zero with a large amplitude of ~46°.76.

In contrast, panel e for the sesqui-synchronous resonance $p = 3/2$ shows that ϕ_* generally decreases by 180° per orbit, but makes large prograde loops with an amplitude of ~22°.67 about $\phi_* = 0$ or ±180° on alternating orbits. Panel f for the $p = 2$ resonance shows that ϕ_* now generally decreases by 360° each orbit, but makes modest prograde loops about $\phi_* = 0$ with an amplitude of ~7°.73.

Panel g of Fig. 1 for the $p = 5/2$ spin–orbit resonance shows that ϕ_* makes tiny prograde loops with an amplitude of only ~0°.17 about $\phi_* = 0$ and ±180° on alternating orbits, during the brief intervals when $dv/dt > 5n/2$. For comparison, panel h for the $p = 3$ resonance shows that ϕ_* decreases monotonically (except when it wraps around the date line), slowing only slightly near $\phi_* = 0$ at perihelion passages.

Panel j for $p = 4$ also shows similar behavior to panel h, while panel i for $p = 7/2$ shows a slight slowing near $\phi_* = \pm180°$ as well as near $\phi_* = 0$ at alternating perihelion passages. Note that all of the panels of Fig. 1 (except panel a) are point-symmetric about the center $nt = 360°$, $\phi_* = 0$.

Compare my Fig. 1 with Fig. 1 of Wang et al. (2014b). For the synchronous state $p = 1$, my panel d agrees with their panel a. For the sesqui-synchronous resonance $p = 3/2$, however, my panel e disagrees slightly with their panel c; their curve is not quite periodic, as spin–orbit resonance requires, but instead rises by