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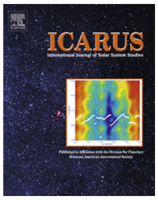

Tides on Europa: The membrane paradigm

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ABSTRACT

Jupiter's moon Europa has a thin icy crust which is decoupled from the mantle by a subsurface ocean. The crust thus responds to tidal forcing as a deformed membrane, cold at the top and near melting point at the bottom. In this paper I develop the membrane theory of viscoelastic shells with depth-dependent rheology with the dual goal of predicting tidal tectonics and computing tidal dissipation. Two parameters characterize the tidal response of the membrane: the effective Poisson's ratio $\bar{\nu}$ and the membrane spring constant Λ, the latter being proportional to the crust thickness and effective shear modulus. I solve membrane theory in terms of tidal Love numbers, for which I derive analytical formulas depending on $\Lambda, \bar{\nu}$, the ocean-to-bulk density ratio and the number k_2° representing the influence of the deep interior. Membrane formulas predict h_2 and k_2 with an accuracy of a few tenths of percent if the crust thickness is less than one hundred kilometers, whereas the error on l_2 is a few percents. Benchmarking with the thick-shell software SatStress leads to the discovery of an error in the original, uncorrected version of the code that changes stress components by up to 40%. Regarding tectonics, I show that different stress-free states account for the conflicting predictions of thin and thick shell models about the magnitude of tensile stresses due to nonsynchronous rotation. Regarding dissipation, I prove that tidal heating in the crust is proportional to $Im(\Lambda)$ and that it is equal to the global heat flow (proportional to $Im(k_2)$) minus the core-mantle heat flow (proportional to $Im(k_2^\circ)$). As an illustration, I compute the equilibrium thickness of a convecting crust. More generally, membrane formulas are useful in any application involving tidal Love numbers such as crust thickness estimates, despinning tectonics or true polar wander.

1. Introduction

The few facts about the interior of Jupiter's moon Europa that really matter for tides come down to a simple formula: 'a thin icy crust floating on a subsurface ocean'. The tidal response of Europa is not unlike a water balloon thrown in the air. The balloon membrane is stretched around the deformed water mass and tries to put it back into its initial shape without much success. Regarding tidal effects, Europa is thus more a 'membrane world' than an 'ocean world' (McKinnon et al., 2009). The term 'membrane paradigm' in the title is, of course, a tongue-in-cheek reference to the black hole model in which a fictitious membrane located just outside the horizon is endowed with conductivity and other physical properties (Price and Thorne, 1988).

The existence of an ocean within Europa is nearly certain since the Galileo spacecraft detected a magnetic induction signature that can only be explained by a near-surface conductive layer, most likely a saline ocean (Khurana et al., 1998; Khurana et al., 2009). Close-up pictures by Galileo also revealed vast chaotic provinces looking like terrestrial pack ice (Carr et al., 1998; Collins and Nimmo, 2009). Furthermore, detailed modeling of tectonic features suggests that they are caused, at least in part, by tidal flexing of a thin floating ice shell (Hoppa et al., 1999b; Kattenhorn and Hurford, 2009). A key prediction of this model was recently verified when Roth et al. (2014) detected water vapor above Europa's south pole at the apocenter of the orbit.

On Europa, tides and ocean are mutually dependent. On the one hand, the subsurface ocean partially decouples the crust from the deep interior and thus increases tidal deformations by a factor of 20 or more, depending on the elasticity of the mantle (Moore and Schubert, 2000; Sotin et al., 2009). On the other, tidal heating within the crust is larger than radiogenic heat from the mantle and is probably necessary to keep the ocean from freezing (Hussmann et al., 2002; Spohn and Schubert, 2003). Tides are thus an essential ingredient in modeling internal structure and thermal evolution. The other important domain of application of tides is the prediction of the numerous tectonic features which are mainly attributed to eccentricity tides, with possible contributions from obliquity tides (plus spin pole precession), physical librations and

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nonsynchronous rotation (Kattenhorn and Hurford, 2009; Rhoden and Hurford, 2013 and references therein).

The role of the ocean in tides is all the more important because the crust is thin (in a sense precised below), with the result that the crust offers little resistance to the changing tidal bulge of the ocean. Gravity data constrain the total water layer thickness (crust plus ocean) to be less than 170 km (Anderson et al., 1998) so that the crust thickness itself must be less than 10% of Europa's radius. Various methods have been applied to infer crust thickness, yielding a wide range of estimates from less than one kilometer to a few tens of kilometers (see reviews by Billings and Kattenhorn (2005), Nimmo and Manga (2009) and McKinnon et al. (2009)). In any case, the highest estimates are no more than a few percents of the surface radius. From the point of view of ocean-surface exchanges, a 20 km-thick crust is certainly not thin. Mechanics, however, require a less stringent criterion: *thin shell theory* is typically considered a good model for the deformation of a shell if its thickness is less than 5–10% of the body's radius (Novozhilov, 1964; Kraus, 1967). This constraint depends on the wavelength of deformations and is thus considerably relaxed for tidal deformations which have a wavelength equal to half the circumference (Beuthe, 2008). If deformations have a very long wavelength compared to shell thickness, thin shell theory takes a simpler form called the *membrane theory of shells*. Confusingly, planetologists call the latter approach the thin shell approximation.

All tidal effects can be predicted by computing deformations of the whole satellite with the theory of viscoelastic-gravitational deformations (e.g. Saito, 1974). The fundamental equations of this theory can be solved in different ways depending on the approximations made: propagation matrix method if incompressible body and static tides (Segatz et al., 1988; Moore and Schubert, 2000; Roberts and Nimmo, 2008; Jara-Orué and Vermeersen, 2011), numerical integration if compressible body and static tides (Wahr et al., 2006; Wahr et al., 2009) or dynamical tides (Tobie et al., 2005). While these codes are in principle accurate, they also have some drawbacks. First, they require a certain expertise, especially if one wants to modify the configuration of the layers (for example adding a fluid core). Second, they are not publicly available except SatStress (Wahr et al., 2009). Third, their results have not yet been systematically compared to each other as it was done for Earth deformations (Spada et al., 2011) so that programming errors remain a possibility. Fourth, codes based on numerical integration typically diverge if tidal frequencies are too low or if solid layers are too soft.

In contrast with the 'black box' approach of viscoelastic-gravitational codes, the membrane theory of elastic shells provides simple analytical formulas for tidal stresses (Vening-Meinesz, 1947). It has thus been very popular to predict tidal tectonic patterns (e.g. Leith and McKinnon, 1996; Greenberg et al., 1998; Kattenhorn and Hurford, 2009). Why not extend it to other applications? The problem with membrane theory in its present form is that it is restricted to an elastic and homogeneous crust. Assuming elasticity makes it impossible to compute viscoelastic tidal deformations and tidal dissipation. Requiring homogeneity is problematic too because the rheology of ice changes with depth. The viscosity of ice sensitively depends on the local temperature of the ice and thus varies by several orders of magnitude between the cold surface and the bottom of the icy shell, where it is at its melting point. Therefore, the elastic thickness of the membrane has a non-trivial relation to the total thickness of the crust, especially if crustal ice is convecting.

In this paper, I extend the membrane theory of shells to viscoelastic shells with depth-dependent rheology. The main goal is to derive ready-to-use formulas for viscoelastic tidal stresses and tidal dissipation. I choose to reformulate the membrane approach in terms of the tidal Love numbers describing the tidal response of the body (Love, 1909), for which I derive analytical formulas in the membrane approximation. Using Love numbers offers three advantages:

(1) Universality: tidal Love numbers appear in many applications for which a theoretical framework already exists. It is unnecessary to develop a parallel formalism in the membrane approach.
(2) Flexibility: the influence of the internal structure can be analyzed by computing the Love numbers for various models without changing the rest of the formalism.
(3) Consistency: the membrane approach clearly appears as a limiting case of the more complete theory of viscoelastic-gravitational deformations. As an illustration, I explain conflicting predictions about the magnitude of nonsynchronous stresses.

Love numbers can be measured with an orbiter (h_2 and k_2, Wu et al. (2001); Wahr et al. (2006)), from multiple flybys (k_2 only, Park et al. (2011)) or with a lander (h_2, l_2 and k_2, Hussmann et al. (2011)). Table 1 gives a list of possible applications of tidal Love numbers, references where formulas in terms of Love numbers can be found, and the sections where the subject is discussed in this paper. The table does not mention one important application: benchmarking numerical codes designed to compute Love numbers and viscoelastic stresses. I will show that membrane formulas are accurate enough to reveal a previously undetected error in the original, uncorrected version of the SatStress code used to predict tidal tectonics (the error is now fixed in the online version).

2. Love numbers in thick shell theory

I will benchmark the membrane approach with analytical and numerical methods based on the theory of viscoelastic-gravitational deformations. This approach is sometimes called 'thick shell theory' when the outer shell is lying on top of a liquid or quasi-fluid layer. Before describing the benchmarks, I will summarize the important features that an interior model of Europa should have regarding tidal deformations.

2.1. Interior structure of Europa

There are only two observational constraints on the interior density: the mean density (see Table 2) and the axial moment of inertia factor (Anderson et al., 1998). Therefore, inferences on the density stratification cannot go beyond two or three layers. Reviewing the constraints on the density structure, Schubert et al. (2009) conclude that Europa has (1) a metallic core having a radius between 13% and 45% of the surface radius, (2) a silicate mantle, and (3) a water ice-liquid outer shell which is 80–170 km thick (the density contrast between ocean and icy shell is unconstrained).

Table 1
Tidal Love numbers: applications.

Topic	h_2	l_2	k_2	Reference	In this paper
Crust thickness	✓		✓	Wahr et al. (2006)	Section 4.4
Tidal tectonics	✓	✓		Wahr et al. (2009)	Section 6.2
Despinning tectonics	✓	✓		Beuthe (2010)	–
Local dissipation	✓	✓		Beuthe (2013)	Section 7.2
Global heat flow			✓	Segatz et al. (1988)	Section 7.4
True polar wander	✓		✓	Matsuyama et al. (2014)	Section 8