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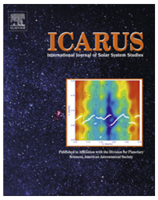

Saturn ring seismology: Evidence for stable stratification in the deep interior of Saturn

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ABSTRACT

Seismology allows for direct observational constraints on the interior structures of stars and planets. Recent observations of Saturn's ring system have revealed the presence of density waves within the rings excited by oscillation modes within Saturn, allowing for precise measurements of a limited set of the planet's mode frequencies. We construct interior structure models of Saturn, compute the corresponding mode frequencies, and compare them with the observed mode frequencies. The fundamental mode frequencies of our models match the observed frequencies (of the largest amplitude waves) to an accuracy of ~1%, confirming that these waves are indeed excited by Saturn's f-modes. The presence of the lower amplitude waves (finely split in frequency from the f-modes) can only be reproduced in models containing gravity modes that propagate in a stably stratified region of the planet. The stable stratification must exist deep within the planet near the large density gradients between the core and envelope. Our models cannot easily reproduce the observed fine splitting of the $m = -3$ modes, suggesting that additional effects (e.g., significant latitudinal differential rotation) may be important.

1. Introduction

The interior structures of planets other than the Earth are generally poorly constrained. Although theoretical studies abound, our understanding is hampered by the lack of direct observational constraints (see Guillot, 2005; Fortney and Nettelmann, 2009; Guillot and Gautier, 2014 for reviews). With thousands of recently discovered exoplanets/exoplanet candidates, a basic understanding of the internal structures of giant planets is more important than ever.

Seismology offers the best hope for directly inferring interior structures of planets. Indeed, our understanding of the Earth's interior owes its existence primarily to seismic measurements. We advise the interested reader to consult Dahlen and Tromp (1998), hereafter DT98, for a comprehensive description of the techniques of Earth seismology. Chaplin and Miglio (2013) presents a review of recent developments in asteroseismology, while Lognonne and Mosser (1993) and Stein and Wysession (2003) discuss results in terrestrial seismology. Unfortunately, seismic measurements of other planets are much more difficult, and no unambiguous detections of oscillations in the outer Solar System planets exist (although there are tentative detections of pressure modes in Jupiter via radial velocity techniques, see (Gaulme et al., 2011) and discussion in Section 6.3).

Saturn provides an amazing opportunity to indirectly detect global oscillation modes through their interaction with Saturn's rings. Marley (1991) (M91) and Marley and Porco (1993) (MP93) predicted that some of Saturn's oscillation modes (in particular the prograde f-modes) could be detected through waves in the rings launched at Lindblad resonances with the gravitational forcing created by the modes (see also Pena, 2010). This prediction was confirmed by Hedman and Nicholson (2013) (HN13), who used *Cassini* data to measure the azimuthal pattern numbers m and pattern frequencies Ω_p of several unidentified waves within Saturn's C Ring. They found that the frequencies and pattern numbers matched Marley & Porco's predictions to within a few percent, and that the waves could not be explained by any other known process.

The puzzling aspect of HN13's findings is that there are *multiple* waves of the same m near the locations predicted by M91 and MP93, whereas only one wave was expected. The multiple waves, split by less than 10% in frequency, appear to be generated by distinct oscillation modes within Saturn whose frequencies are split by the same fractions. The observed splitting is not simple rotational splitting (as this occurs between oscillation modes of different m) and suggests more complex physics is at play.

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Fuller et al. (2014) (F14) investigated the effect of a solid core on the oscillation mode spectrum of Saturn. They found that if Saturn has a large solid core that is relatively unrigid (has a small shear modulus μ), the shear oscillation modes of the core can exist near the same frequencies as the f-modes that generate some of the observed waves in the rings. Modes very close in frequency to the f-modes can degenerately mix with them (a process also known as avoided crossing), attaining large enough gravitational potential perturbations to generate waves in the rings. However, F14 found that degenerate mixing was rare, and that only finely tuned models could qualitatively reproduce the observed mode spectrum. The oscillations of rotating giant planets have also been examined in several other works (Vorontsov and Zharkov, 1981; Vorontsov, 1981, 1984, M91; Wu, 2005; Pena, 2010; Le Bihan and Burrows, 2012; Jackiewicz et al., 2012; Braviner and Ogilvie, 2014). None of these works have extensively examined the effect of stable stratification and the resulting planetary mode spectrum (although M91 does briefly consider the effect of stable stratification on the f-mode frequencies).

In this paper, we examine Saturn's oscillation mode spectrum in the presence of a stably stratified region deep within the planet. Regions of stable stratification have been speculated to exist within giant planets due to the stabilizing effect of composition gradients (Leconte and Chabrier, 2013). The composition gradients could be produced by dissolution of heavy core elements in the helium/hydrogen envelope (Wilson and Militzer, 2012a,b) or by gravitational settling of metals (Stevenson, 1985) or helium (Salpeter, 1973; Stevenson and Salpeter, 1977). Recent simulations have sought to determine the large-scale time evolution of doubly diffusive convection produced by competing thermal/composition gradients (Rosenblum et al., 2011; Mirouh et al., 2012; Wood et al., 2013), but the resulting global structure of giant planets is unclear. Fig. 1 shows a simple schematic of the type of Saturn models we consider. It should not be interpreted too strictly, it is intended only to provide the reader with a general picture of our hypothesis for Saturn's interior structure.

If stably stratified regions exist, they allow for the existence of gravity modes (g-modes) in the oscillation mode spectrum. For stable stratification deep within the planet, the g-modes can exist in the same frequency regime as the f-modes and can strongly mix with them. This process is analogous to the mixed g-modes/p-modes observed in red giant stars, although somewhat complicated by Saturn's rapid rotation. Our calculations reveal that g-mode mixing can naturally explain the observed splitting between the $m = -2$ waves, but cannot robustly reproduce the fine splitting between the $m = -3$ waves. We claim this is strong evidence for the existence of stable stratification within Saturn, although some important physical ingredient (e.g., differential rotation) may be required for a complete understanding.

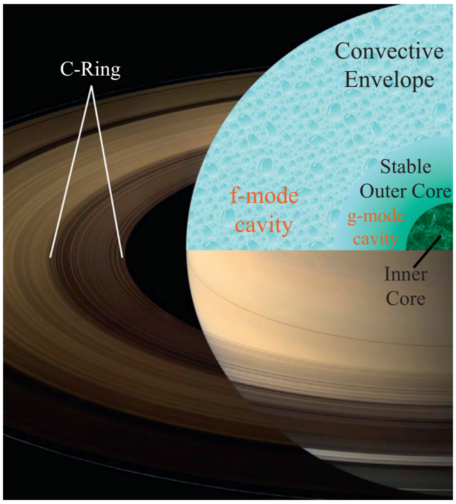

Fig. 1. *Cassini* image of Saturn and its rings, overlaid with a schematic cartoon of our hypothesis for Saturn's interior structure. The structure shown here is not quantitatively accurate. It is meant only to illustrate the general features of Saturn's interior structure that we advocate: a thick convective envelope (which harbors f-modes, p-modes, and i-modes) overlying a region of stable stratification near the core of the planet (which harbors g-modes and r-modes). We have also pointed out the C-ring, where all of the mode-driven waves of been observed.

Our paper is organized as follows. Section 2 describes the toy Saturn models we use in our calculations. Section 3 summarizes our method of solving for oscillation modes in the presence of rapid rotation, and reviews the types of modes that exist in rotating planets. In Section 4 we examine the process of mode mixing induced by rotation, centrifugal, and ellipticity effects, and describe how this affects mode frequencies and eigenfunctions. Section 5 compares our results to observations, and we conclude with a discussion of these results in Section 6. This section also addresses the issues of mode amplitudes, mode driving, and the prospects for observing saturnian and jovian p-modes via radial velocity techniques.

2. Saturn models

The interior structure of giant planets is not particularly well-constrained. Other than its mass M and radius R, the strongest observational constraint on Saturn's interior structure is the measured value of the gravitational moment J_2, which indicates that Saturn must have a dense core of $\sim 15\,M_\oplus$ (Guillot, 2005). We therefore create toy models which roughly match the measured values of M, equatorial radius $R_{\rm eq}$, polar radius $R_{\rm po}$, and J_2. We do not attempt to rigorously compare these models with any theoretical equations of state or microphysical models, although in Section 6 we discuss how our models relate to recent theoretical developments in planetary interiors.

For the purposes of our adiabatic mode calculations, the only physical quantities of importance in Saturn's interior are the density ρ, Brunt–Vaisala frequency N, sound speed c_s, gravitational acceleration g, and spin frequency Ω_s. To create a toy model, we proceed as follows. We first create a spherical model with a polytropic density profile of index $(n = 1)$, with a density profile $\rho_1(r)$. We choose a sound speed c_{s1} such that the Brunt–Vaisala frequency

$$N_1^2 = -g_1 \frac{d\ln\rho_1}{dr} - \frac{g_1}{c_{s1}^2} \tag{1}$$

is equal to zero everywhere.

We then choose inner and outer core radii $r_{\rm in}$ and $r_{\rm out}$, and a core density enhancement D. We multiply the density of the inner core by D, such that $\rho(r) = D\rho_1(r)$ for $r < r_{\rm in}$. The density of the outer core is calculated via

$$\rho(r) = \rho_1(r)\Big[1 + (D-1)\sin^2[(\pi/2)(r_{\rm out} - r)/(r_{\rm out} - r_{\rm in})]\Big] \quad \text{for} \quad r_{\rm in} < r < r_{\rm out} \tag{2}$$

This form is somewhat arbitrary; we use it to obtain a smooth density increase between the envelope and inner core. In the outer core, we readjust the soundspeed such that

$$c_s^2(r) = c_{s1}^2(r_{\rm out}) + \big[c_{s1}^2(r_{\rm in}) - c_{s1}^2(r_{\rm out})\big]\sin[(\pi/2)(r_{\rm out} - r)/(r_{\rm out} - r_{\rm in})] \quad \text{for} \quad r_{\rm in} < r < r_{\rm out}. \tag{3}$$

Once again, this sound speed profile is somewhat arbitrary. This form ensures a positive value of N^2 in the outer core. Because we focus only on f-modes (for which ρ is the defining quantity) and