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A dynamo model of Jupiter's magnetic field

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ABSTRACT

Jupiter's dynamo is modelled using the anelastic convection-driven dynamo equations. The reference state model is taken from French et al. [2012]. Astrophys. J. Suppl. 202, 5, (11pp), which used density functional theory to compute the equation of state and the electrical conductivity in Jupiter's interior. Jupiter's magnetic field is approximately dipolar, but self-consistent dipolar dynamo models are rather rare when the large variation in density and the effective internal heating are taken into account. Jupiter-like dipolar magnetic fields were found here at small Prandtl number, $Pr = 0.1$. Strong differential rotation in the dynamo region tends to destroy a dominant dipolar component, but when the convection is sufficiently supercritical it generates a strong magnetic field, and the differential rotation in the electrically conducting region is suppressed by the Lorentz force. This allows a magnetic field to develop which is dominated by a steady dipolar component. This suggests that the strong zonal winds seen at Jupiter's surface cannot penetrate significantly into the dynamo region, which starts approximately 7000 km below the surface.

1. Introduction

Jupiter has the strongest magnetic field of any planet in the Solar System (Connerney, 1993). It is believed to be generated by convection-driven flows in the metallic hydrogen region of the planet (Parker, 1979; Stevenson, 1983, 2003; Jones, 2011). As the planet gradually cools, convection rather than radiation carries the heat flux out (Guillot et al., 2005), leading to an equilibrium reference state which is close to adiabatic. To model the dynamo, we use the self-consistent convection-driven dynamo equations in the anelastic approximation (Braginsky and Roberts, 1995; Lantz and Fan, 1999), which takes into account the large variation in density with depth. The model is based on an equilibrium reference state which uses an equation of state derived from density functional theory, and the electrical conductivity used here is also based on *ab initio* calculations (French et al., 2012). The model contains a small rocky core releasing less than 2% of Jupiter's intrinsic heat flux. As Jupiter cools, it releases an approximately uniform specific entropy everywhere outside the core, so the driving is different from the geodynamo, where the main buoyancy source is believed to be near the inner core boundary (basal heating).

In Boussinesq convection-driven dynamos, dipolar solutions occupy a large region of the numerically accessible parameter space (e.g. Olson et al., 1999; Jones, 2011). Strong dipolar dominance is found for low E/Pm and moderate Rm, where E is the Ekman number, Pm the magnetic Prandtl number, and Rm the magnetic Reynolds number. Here dynamos are even more dipolar than the geomagnetic field (Christensen et al., 2010).

Jupiter's magnetic field is approximately dipolar, but strongly dipolar solutions for anelastic dynamos with large density ratios across the convecting shell are much harder to find (Gastine et al., 2012), a result confirmed here. Polytropic reference state models with uniform electrical conductivity only give dipolar solutions for density ratios less than 5 at $Pr = 1$ (Gastine et al., 2012). At high density ratios the convective velocities are largest near the surface both in the linear (Glatzmaier and Gilman, 1981; Jones et al., 2009) and nonlinear (Jones and Kuzanyan, 2009) regimes, and this enhances the helicity and zonal flow in the outer regions, leading to large small-scale fields there which dominate the dipolar component (Gastine et al., 2012). When the low electrical conductivity region in the non-metallic outer zone is taken into account, dipolar dynamos have been found in Boussinesq (Gómez-Pérez et al., 2010) and polytropic models (Duarte et al., 2013), because the strong convection beyond the transition zone no longer generates disruptive small-scale fields. However, it was generally found that the transition zone between the electrically conducting region and the molecular insulating region must be in the range 0.7 to $0.8r_{jup}$ (Duarte et al., 2013) to get dipolar solutions. The recent *ab initio* calculations (French et al., 2012) suggest the transition zone is further out at $\sim 0.9r_{jup}$. These polytropic models were driven by basal heating; the more realistic uniform entropy source models compound the difficulty by enhancing the

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convection in the outermost layers. Simulations at low E and $Pr = 0.03$ were performed by Glatzmaier (Stanley and Glatzmaier, 2010) for a Jupiter model: dipolar fields were obtained, though the generated field pattern was not very Jupiter-like.

The new features of this model compared with previous anelastic work (Gastine et al., 2012; Duarte et al., 2013) are (i) the reference state model is based on a Jupiter model (French et al., 2012) rather than a polytropic state; this reference state is broadly similar to other existing models (Hubbard, 1968; Hubbard and Marley, 1989; Guillot, 1999). (ii) there is a uniform specific entropy source rather than basal heating; (iii) a different range of parameter space was explored, in particular the Prandtl number was varied and more strongly driven models were investigated. Basal heating may be appropriate for geodynamo models where compositional convection is occurring, but in Jupiter the bulk of the heat flux comes from the cooling of the hydrogen/helium envelope and not from the small core. Many different runs of this model, some with a combination of internal and basal heating, were performed, but only a few are discussed in detail here. Stable dipolar solutions were found when basal heating dominates, as also found by Duarte et al. (2013), but with a uniform specific entropy source, dipole dominated solutions were found only at low Prandtl number, so this is where our results are focussed. The anelastic version of the Leeds dynamo code (see e.g. Gubbins et al., 2007) was used, which has passed the anelastic dynamo benchmark test (Jones et al., 2011). The simulations required substantial computational resources. Most runs confirmed the view that dipolar runs are hard to find (Gastine et al., 2012; Duarte et al., 2013). While the precise form of the convection and magnetic field patterns do depend on the reference state model, the switch to the new reference state does not appear to change the general picture dramatically, because our runs with basal heating gave similar results to those of Duarte et al. (2013), who used a polytropic model. As expected, the switch to a uniform entropy source makes dipolar fields even harder to find. The enhanced convection in the outer regions generated more magnetic activity there (Gastine et al., 2012; Duarte et al., 2013), and this activity is typically small-scale and cannot co-exist with a dipolar field. Consistent with previous work (Gastine et al., 2012) we found small-scale dynamos, hemispherical dynamos (Grote and Busse, 2000) in which the generated field is predominantly in one hemisphere, and Parker dynamo waves (Parker, 1979) in various regions of the parameter space. It is not computationally possible to extend the model to the surface of Jupiter, because the convection becomes very small-scale near the surface, demanding very small time-steps and a massive resolution requirement. The model has therefore been cut off at $r = r_{cut}$, 3000 km below the surface, above which the electrical conductivity is essentially zero.

2. Equations of the model

The Lantz–Braginsky–Roberts anelastic dynamo equations (Braginsky and Roberts, 1995; Lantz and Fan, 1999; Jones et al., 2011) were used in a spherical shell between the core radius r_c and the cut-off radius r_{cut}.

The usual form of the equation of motion in dimensional form is (e.g. Jones et al., 2011),

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} = \frac{1}{\bar\rho}\mathbf{j}\times\mathbf{B} - 2\boldsymbol{\Omega}\times\mathbf{u} + \mathbf{F}_v - \frac{\nabla p'}{\bar\rho} - \nabla\Phi' - \frac{\rho'\nabla\bar\Phi}{\bar\rho}, \tag{2.1}$$

and a perfect gas is often assumed, but the metallic hydrogen in the dynamo region of Jupiter means that the gas is far from perfect. Here $\mathbf{u}$ is the velocity in the rotating frame, $\mathbf{j}$ is the current density, $\mathbf{B}$ is the magnetic field, $\boldsymbol{\Omega}$ is the rotational angular velocity, p is the gas pressure (including that coming from electron degeneracy in the metallic hydrogen region), ρ is the density and Φ the gravitational potential. The equilibrium state is assumed to be spherically symmetric for simplicity, but no symmetry is assumed for the disturbances (denoted by primes) produced by the convection. As usual in the anelastic approximation these disturbances are assumed not to alter the equilibrium density and pressure significantly (e.g. Lantz and Fan, 1999), and since the convective velocity in Jupiter is always much less than the sound speed this is a reasonable assumption. Hence the gravitational acceleration of the equilibrium state is $\mathbf{g} = -g\hat{\mathbf{r}} = -\nabla\Phi$ and Φ is decomposed into $\bar\Phi + \Phi'$. We now put this equation into Lantz–Braginsky–Roberts form without assuming a perfect gas: see also (Ingersoll and Pollard, 1982; Braginsky and Roberts, 1995; Kaspi et al., 2009). Define $\hat p = p'/\bar\rho + \Phi'$, so

$$-\frac{\nabla p'}{\bar\rho} - \nabla\Phi' - \frac{\rho'}{\bar\rho}\nabla\bar\Phi = -\nabla\hat p - \hat{\mathbf{r}}\left[\frac{p'}{\bar\rho^2}\frac{d\bar\rho}{dr} + \frac{\bar g\rho'}{\bar\rho}\right], \tag{2.2}$$

overbars denoting equilibrium state values. Now

$$\rho' = \left(\frac{\partial\rho}{\partial S}\right)_p S' + \left(\frac{\partial\rho}{\partial p}\right)_S p' = \left(\frac{\partial\rho}{\partial S}\right)_p S' + \left(\frac{d\bar\rho}{d\bar p}\right)p', \tag{2.3}$$

since the basic state is close to adiabatic, so making use of the hydrostatic equation $d\bar p/dr = -g\bar\rho$,

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} = \frac{1}{\bar\rho}\mathbf{j}\times\mathbf{B} - 2\boldsymbol{\Omega}\times\mathbf{u} + \mathbf{F}_v - \nabla\hat p - \hat{\mathbf{r}}\frac{gS'}{\bar\rho}\left(\frac{\partial\rho}{\partial S}\right)_p. \tag{2.4}$$

We can rewrite this in a more useful form using Maxwell's thermodynamic relations. The enthalpy $\mathcal{H} = U + p/\rho$ can be expressed in differential form as $d\mathcal{H} = TdS + dp/\rho$.. So

$$\left(\frac{\partial\mathcal{H}}{\partial S}\right)_p = T, \quad \left(\frac{\partial\mathcal{H}}{\partial p}\right)_S = \frac{1}{\rho}, \quad \text{so} \left(\frac{\partial T}{\partial p}\right)_S = \frac{\partial^2\mathcal{H}}{\partial S\partial p} = -\frac{1}{\rho^2}\left(\frac{\partial\rho}{\partial S}\right)_p. \tag{2.5}$$

Again using the hydrostatic equation and the fact that the reference state is close to adiabatic,

$$\frac{g}{\bar\rho}\left(\frac{\partial\rho}{\partial S}\right)_p = -g\bar\rho\left(\frac{\partial T}{\partial p}\right)_S = \frac{d\bar p}{dr}\left(\frac{d\bar T}{d\bar p}\right), \tag{2.6}$$

so the equation of motion in final form is

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} = \frac{1}{\bar\rho}\mathbf{j}\times\mathbf{B} - 2\boldsymbol{\Omega}\times\mathbf{u} + \mathbf{F}_v - \nabla\hat p - \hat{\mathbf{r}}\frac{d\bar T}{dr}S'. \tag{2.7}$$

The equations are scaled using the units

$$t = \frac{d^2}{\eta_m}t^*, \quad \nabla = \frac{1}{d}\nabla^*, \quad \text{where } d = r_{cut} - r_c, \quad \bar\rho = \rho_m\bar\rho^*, \tag{2.8}$$

$$\mathbf{B} = \sqrt{\Omega\rho_m\mu_0\eta_m}\mathbf{B}^*, \quad S' = \Delta S S^*, \quad T = T_m T^*, \tag{2.9}$$

where $r_{cut} = 6.7\times10^7$ m is the cut-off radius, r_c is the core radius, and the subscript m denotes values at the midpoint $r = r_m = (r_c + r_{cut})/2$. The small entropy drop across the layer is ΔS. The equation of motion (2.7) becomes, dropping the *,

$$\frac{1}{Pm}\left(\frac{\partial\mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla\frac{\hat p}{E} + \left[-\frac{2}{E}\hat{\mathbf{z}}\times\mathbf{u} + \frac{1}{E\bar\rho}(\nabla\times\mathbf{B})\times\mathbf{B} + \mathbf{F}_v - \frac{Pm}{Pr}RaS\frac{d\bar T}{dr}\hat{\mathbf{r}}\right]. \tag{2.10}$$

The entropy equation is

$$\frac{\mathcal{D}S}{\mathcal{D}t} = \frac{Pm}{Pr}\left(\frac{1}{\bar\rho\bar T}\nabla\cdot\bar\rho\bar T\nabla S + H\right) + \frac{Pr}{PmRa\bar T}\left[\frac{\bar\eta}{E\bar\rho}(\nabla\times\mathbf{B})^2 + Q_v\right], \tag{2.11}$$