Icarus 234 (2014) 194–199

Contents lists available at ScienceDirect

Icarus

journal homepage: www.elsevier.com/locate/icarus

Note

Saturn ring seismology: Looking beyond first order resonances

Mark S. Marley

NASA Ames Research Center, Mail Stop: 245-3, P.O. Box 1, Moffett Field, CA 94035-0001, United States

ARTICLE INFO

Article history:
Received 5 December 2013
Revised 31 January 2014
Accepted 2 February 2014
Available online 14 February 2014

Keywords:
Saturn, Rings
Saturn, interior
Saturn
Resonances, rings

ABSTRACT

Some wave features found in the C-ring of Saturn appear to be excited by resonances with normal mode oscillations of the planet. The waves are found at locations in the rings where the ratio of orbital to oscillation frequencies is given by $m : m+1$ where m is a small integer. I suggest here that it is plausible that ring waves may also be launched at second order resonances where the frequency ratio would be $m : m+2$. Indeed otherwise unassociated wave features are found at such locations in the C-ring. If confirmed the association of planetary modes with additional C-ring wave features would measure additional oscillation frequencies of Saturn and improve the utility of the waves for constraining the internal structure of the planet. Second-order resonances in general do not lie near first order ring resonance locations and thus are not the explanation for the apparent frequency splitting of modes.

Published by Elsevier Inc.

1. Introduction

Marley (1990) and Marley and Porco (1993) proposed that certain wave features in Saturn's C-ring discovered by Rosen et al. (1991), as well as the Maxwell gap, were created by resonant interaction with internal oscillation modes of Saturn. Since the oscillation modes perturb the internal density profile and consequently the external gravitational field of the planet, they have the potential to excite vertical and horizontal excursions in ring particle orbits analogously to the forcing applied by a satellite of the planet. The precise radial location of the ring features depends on the planetary oscillation mode frequency and thus the rings can serve as a seismometer, recording the frequencies of saturnian oscillations.

While Marley and Porco (1993) argued that the Rosen waves could be associated with saturnian oscillation modes,[1] their detailed predictions for the characteristics of the waves expected to be excited in the rings could not be tested by the Voyager data available at the time. In particular the Voyager Radio Science dataset employed by Rosen et al. (1991) (hereafter R91) was not adequate to precisely ascertain the azimuthal wavenumber, m, of the ring features beyond the crude constraint that m was generally small, in the range of 2–5 or so. Furthermore while the direction of propagation of the waves was apparent, the Voyager data were not sufficient to ascertain whether the R91 waves were density or bending waves.

Such details were important since waves launched by interactions with saturnian internal modes would behave differently than waves launched by external satellites. Density waves launched at inner Lindblad resonances (ILR) with external satellites propagate outwards – away from Saturn – while bending waves launched at inner vertical resonances (IVR) propagate inwards. If any of the wave features identified in R91 were indeed produced by resonances with the planet, then the direction of propagation would be the opposite of the satellite case. Bending waves should propagate outwards and density wave inwards. Since neither the wave type (bending or density) nor their azimuthal wavenumber were constrained by R91, the ring waves could thus not be definitively associated with oscillation modes of the planet. The situation did not change for 20 years.

Recently, however, Hedman and Nicholson (2013) used stellar occultation data obtained by the Cassini Visual and Infrared Mapping Spectrometer to place new constraints on the C ring waves first identified by R91. They focused on six waves, four of which had first been identified by R91, and concluded that they were density – not bending – waves and that their azimuthal wavenumbers were consistent with the predictions of Marley and Porco (1993) (hereafter MP93). Hedman and Nicholson (2013) thus concluded that a seismological origin for these waves was likely, thereby confirming the MP93 hypothesis. One surprise, however, was that there were multiple waves with the same azimuthal wavenumber m at different locations in the C-ring, whereas only one wave would be expected. The usual mode splittings arising from rotation and planetary oblateness has already been accounted for in the mode frequency calculation of Marley (1991). Thus, some other property

[1] Marley et al. (1987) originally suggested that Saturn f-modes might launch C-ring waves prior to the announcement of the discovery of unassociated waves by Rosen et al. (1988).

http://dx.doi.org/10.1016/j.icarus.2014.02.002
0019-1035/Published by Elsevier Inc.

of Saturn must permit multiple oscillation modes, each with the same azimuthal wave number, but with slightly different frequencies.

Fuller et al. (2014) examined the possible role of mixing between the global f-mode oscillations of Saturn and modes that might be trapped in Saturn's core. They found that such mixing could in principle create multiple modes with similar frequencies near a single f-mode and explain the apparent mode splitting, but that a substantial 'tuning' of the Saturn envelope and core model would be required to produce the observed ring features and they deemed this unlikely. The origin of this "fine" mode splitting thus remains unknown.

In addition to the waves studied by Hedman and Nicholson (2013), Baillié et al. (2011) also searched Cassini UVIS data for wave features. By summing multiple stellar occultation profiles they could find waves with lower amplitudes than previous studies. They tabulated a total of 40 C-ring wave features, of which only five were associated with possible satellite resonances. With six of the waves apparently of seismic origin a remainder of about 30 could not be explained. The ring wave features tabulated by Baillié et al. (2011) that are not associated with known satellite resonances are plotted in Figs. 1 and 2. Fig. 1 shows those features which appear to propagate inwards, towards Saturn while Fig. 2 shows those that propagate in the opposite direction.

Given the discoveries by Hedman and Nicholson (2013) and Baillié et al. (2011) it seems appropriate to revisit the seismological connection of saturnian oscillation modes with ring wave features with the aim of testing if planet-ring system resonances beyond those considered by MP93 could account for either the observed apparent fine splitting of modes or any of the waves tabulated by Baillié et al. (2011). To address these issues, here I briefly review the theory presented in MP93 and point out that second order resonances can produce many more additional resonant features in the C-ring than were tabulated in MP93.

2. Basics of ring seismology

The theory of planetary mode ring seismology is laid out in detail in MP93. This section provides a very brief summary of the planetary oscillation modes, ring wave excitation mathematics, and explains the calculation of locations of waves launched by a given planetary oscillation mode.

2.1. Which modes?

Several different types of planetary oscillation modes, including p-, g-, and f-modes, which differ in the nature of their restoring force (see Unno et al. (1979)), could in principle launch waves in the rings. A single planetary oscillation mode as observed from inertial space has frequency commonly denoted by $\sigma_{\ell mn}$. The three integers, m, ℓ, and n uniquely identify a specific mode.[2] The index ℓ enumerates the number of circles bounding regions of positive or negative perturbation at the surface; $\ell - |m|$ is equal to the number of boundaries in latitude. By the convention used here modes propagating in the same (opposite) direction of rotation of the planet have positive (negative) values of m. The number of radial nodes from the surface to the center of the planet is denoted by n.

As described in MP93 the modes most likely to produce features in the C-ring are f-mode oscillations which are defined as those with no radial nodes, or $n = 0$. This is because the perturbation to the external gravitational field will be greatest when the density perturbation inside the planet is in phase from surface to core. When $n > 0$ the radial density perturbation is no longer in phase from core to atmosphere, resulting in a reduced perturbation to the external gravitational field of the planet. In addition, the frequencies of the $n > 0$ modes are too high to resonate with ring particle orbits. As ℓ increases the f-modes are trapped progressively closer to the surface where the background density is smaller. A given mode amplitude at the surface thus produces progressively smaller perturbations to the external gravitational field as ℓ is increased. Ring seismology thus favors modes of low ℓ.

A complete calculation of the frequency of a planetary oscillation mode requires computing $\sigma^0_{\ell mn}$, the mode frequency in the frame of the planet, as well as corrections for rotation to give the frequency $\sigma_{\ell mn}$ as observed in inertial space. The correction must account both for the change out of Saturn's rotating frame (rotation frequency Ω_{Sat}) and the influence of rotation on the modes themselves. For slow rotation the former correction is simply $m\Omega_{Sat}$ and the latter is typically given as $C_{\ell n}$ such that the total correction is written as

$$\sigma_{\ell mn} = \sigma^0_{\ell mn} + m(1 - C_{\ell n})\Omega_{Sat}. \tag{1}$$

Thus solely because of the frame shift a mode rotating prograde ($m > 0$) will appear to an observer at rest to have a higher frequency than to an observer in the rotating frame. Both Jupiter and Saturn rotate so rapidly, however, that the planet is oblate and a more complex rotation correction – to at least second order in Ω_{Sat} – must be applied (Vorontsov and Zharkov, 1981). However Eq. (1) is conceptually adequate for discussion purposes.

Note that in principle a mode propagating in the opposite direction of Saturn's rotation ($m < 0$) could be carried prograde if $\sigma^0_{\ell mn}$ was slow enough. In practice however $\sigma^0_{\ell mn} > \Omega_{Sat}$ and those modes propagating opposite to Saturn's direction of rotation are never carried prograde by rotation and thus cannot resonantly interact with prograde orbiting ring particles. Because of the periodicity of the oscillation perturbation in azimuth, the pattern frequency of the perturbation Ω_{pat} as seen in inertial space is equal to $\sigma_{\ell mn}/m$. It can sometimes be helpful to think of a mode's "pattern period", $P^{\ell mn}_{pat}$, and these are tabulated in Table 1.

Launching a density wave in the rings requires that there be azimuthal variations in the external gravitational field around the planet as sensed at the equator. By the properties of spherical harmonics this means that only those planetary oscillation modes with either $m = \ell$ or $\ell - |m|$ equal to an even integer can excite density waves in the rings. Other modes will not produce a horizontally varying gravitational potential at the equator. In contrast outer vertical resonances require a vertical gradient in gravitational potential at the equator which in turn implies that $\ell - |m|$ is an odd integer.

In summary if the oscillation modes of Saturn are to launch waves in the rings, the most plausible modes are low ℓ f-modes ($n = 0$) with $m > 0$. But where in the rings will the resonances be found?

2.2. Resonance locations

Given these practical constraints, MP93 derived the expected location for eccentric resonances between planet modes and ring particle orbits. They found that the orbital frequencies Ω associated with Lindblad resonances were given by

$$\frac{\Omega_{pat}}{\Omega} = 1 \mp \frac{q}{|m|}\frac{\kappa}{\Omega}. \tag{2}$$

Here Ω and κ are the orbital angular and epicyclic frequencies of a ring particle at a given location in the rings and q is a positive integer. Outer (inner) resonances take the lower (upper) sign. In the

[2] Strictly speaking the index ℓ defines a spherical harmonic while the planetary modes on a rotating planet consist of a superposition of several spherical harmonics (see discussion in Section 4.2). For simplicity we here equate a single f-mode to a single spherical harmonic.