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Morphology driven density distribution estimation for small bodies

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ABSTRACT

We explore methods to detect and characterize the internal mass distribution of small bodies using the gravity field and shape of the body as data, both of which are determined from orbit determination process. The discrepancies in the spherical harmonic coefficients are compared between the measured gravity field and the gravity field generated by homogeneous density assumption. The discrepancies are shown for six different heterogeneous density distribution models and two small bodies, namely 1999 KW4 and Castalia. Using these differences, a constraint is enforced on the internal density distribution of an asteroid, creating an archive of characteristics associated with the same-degree spherical harmonic coefficients. Following the initial characterization of the heterogeneous density distribution models, a generalized density estimation method to recover the hypothetical (i.e., nominal) density distribution of the body is considered. We propose this method as the block density estimation, which dissects the entire body into small slivers and blocks, each homogeneous within itself, to estimate their density values. Significant similarities are observed between the block model and mass concentrations. However, the block model does not suffer errors from shape mismodeling, and the number of blocks can be controlled with ease to yield a unique solution to the density distribution. The results show that the block density estimation approximates the given gravity field well, yielding higher accuracy as the resolution of the density map is increased. The estimated density distribution also computes the surface potential and acceleration within 10% for the particular cases tested in the simulations, the accuracy that is not achievable with the conventional spherical harmonic gravity field. The block density estimation can be a useful tool for recovering the internal density distribution of small bodies for scientific reasons and for mapping out the gravity field environment in close proximity to small body's surface for accurate trajectory/safe navigation purposes to be used for future missions.

1. Introduction

Scientific interest in small Solar System bodies has been growing significantly during the last decade, with a number of mission studies, actual missions, and planned future missions in the works. In order to support the close proximity operations of the mission, it is necessary to develop an accurate gravity field valid to the surface. One way to construct the surface gravity field is to assume that the body density is homogeneous and compute the potential/acceleration of the spacecraft by shape model integration (Werner and Scheeres, 1997). However, for the majority of asteroids, this homogeneous density assumption may not be suitable, and the measured gravity field from orbit determination process often differs from that generated by shape model integration.

Another gravity field representation that is in wide use is the exterior gravity field (Werner, 2010; Takahashi and Scheeres, 2012; Takahashi et al., 2013). However, it is a well-known fact that the exterior gravity field in Eq. (1) breaks down when the spacecraft penetrates the Brillouin sphere (i.e., circumscribing sphere) of the asteroid:

$$U^e = \frac{GM^*}{R^*}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\left(\frac{R^*}{r}\right)^{n+1}P_{nm}(\sin\phi)\begin{bmatrix}\cos(m\lambda)\\ \sin(m\lambda)\end{bmatrix}\cdot\begin{bmatrix}C^e_{nm}\\ S^e_{nm}\end{bmatrix} \tag{1}$$

where U is the potential, e superscript denotes the exterior quantity, G is the gravitational constant, M^* is the reference mass, R^* is the reference radius, r is the spacecraft position, P_{nm} is the Legendre function of degree n and order m, C_{nm} and S_{nm} are spherical harmonic coefficients, λ is longitude, and ϕ is latitude in the body-fixed frame. There are alternatives to the exterior gravity field model,

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with distinct problems inherent in their expressions: the polyhedral gravity field (i.e., shape model gravity field) (Werner and Scheeres, 1997) assumes a known density distribution, the estimation of which is convoluted; mascon model (mass concentration) is inaccurate near the surface of the body and estimation of each mass element renders a singularity as the number of particles is increased (Park et al., 2010); the conventional expressions of the exterior gravity field and ellipsoidal gravity field fail within the Brillouin sphere/ellipsoid; and the interior gravity field, which is the mirror image of the exterior gravity field, requires an accurate source gravity field (Werner, 2010; Takahashi and Scheeres, 2012).

Had the shape and the density distribution of the body been known, the polyhedral gravity field conveniently maps out the surface gravity field. The polyhedral gravity field contains information equivalent to infinitely many degree and order gravity field expansion of the exterior gravity field. Moreover, the convergence of the potential is guaranteed anywhere on the surface of the body, which is an attractive feature to leverage for proximity operations. However, the accuracy of the polyhedral gravity field depends on the resolution of the shape model and the accuracy of the prescribed density distribution. In general, the shape model can be determined accurately through optical measurements, but the density distribution cannot be uniquely determined. As the gravitation of a body can be modeled error-free had we obtained the error-free shape model and internal density distribution, it is worthwhile to attempt to achieve such a model for scientific reasons as well as for accurate trajectory design/safe navigation purposes. We will focus on the estimation of the density distribution as the shape model is usually determined with sufficient accuracy in the early phase of spacecraft rendezvous, as mentioned above.

Many researchers attempted to tackle the density estimation problem in the past, and our effort should be considered in the same context. For example, Takahashi and Scheeres (2011) have addressed how to identify the heterogeneity in the body density by looking at discrepancies in the spherical harmonic coefficients between a homogeneous body and orbit determination (OD) solution. The discrepancies in the spherical harmonic coefficients between the two models form a measurement that can be used to detect inhomogeneity. Also, they discussed the density estimation technique in an informal conference report (Takahashi and Scheeres, 2013) by leveraging the results of Scheeres et al. (2000), where the density distribution was estimated from the spherical harmonic coefficients determined from OD. Zuber et al. (2000) showed the center-of-mass (COM) and center-of-figure (COF) offset of 433 Eros can be explained by small variations in Eros' internal mechanical structure, given the global homogeneity in surface composition. Park et al. (2010) discussed the density estimation algorithm for mason models with spheres and cubes. The density values of each sphere and cube are estimated by processing the range and range-rate observations. Their results revealed inherent difficulty of the density estimation for a mascon model, where particles close to each other become indistinguishable from one particle of the same total mass depending on the trajectory of the spacecraft, yielding higher uncertainties for the particles placed inward (i.e., farther from the spacecraft) and smaller uncertainties for those placed near the surface of the body (i.e., closer to the spacecraft). In addition, Dawn spacecraft of National Aeronautics and Space Administration (NASA) is en route to a dwarf planet 1 Ceres for a rendezvous in 2015 after it departed from the first target asteroid 4 Vesta in September 2012. Recent effort by Russell et al. (2012) and Asmar et al. (2013) discussed that their core–mantle–crust model produces the second-degree spherical harmonic coefficients that are consistent with the measured gravity field of Vesta.

In this paper, a generalized density estimation algorithm is investigated in order to produce an accurate gravity field model near the surface of the body. As the preliminary analysis, several density distributions are modeled to characterize the relationship between the density distributions and the spherical harmonic coefficients. Then, these characteristics are compiled to make an archive that can be used to cross-correlate the spherical harmonic coefficients determined from OD to the possible density distributions (i.e., both density map and density value, to be discussed later).

The density distribution models investigated in this paper are the planar division, surface layer, single core at the coordinate center, double core placed along the x-axis, a torus around the equator, a cylinder with rings, and blocks. The block model forms the foundation of a new approach to estimating the density distribution, namely the block density estimation. For the block density estimation, a body is dissected into a number of blocks, and the density in each block is estimated by fitting to the spherical harmonics in a least-squares sense. This approach is analogous to filling the body with mascons and is indeed motivated by work by Park et al. (2010). However, the block estimation has significant advantage over the mascons, as it alleviates us from choosing a specific density map to estimate the density value and allows us to control the resolution of the density map throughout the volume more concisely. The results show that an accurate, consistent density distribution map is achieved by increasing the number of blocks.

It is important to note that the *true* density distribution cannot be uniquely and unequivocally determined for *any* density estimation method from the gravimetry data. The density estimation is inherently a non-invertible problem, and at best one can ascribe the best-fit density values to a finite number of density map components. If the density distribution actually follows the assumed density map, then it would be expected that as the number of harmonics is increased the fit may be improved. However, this can never be used as a confirmation that the assumed density map is correct, as there are infinitely many different density distributions that satisfy a given gravity field, as detailed in Section 2. Our approach is not free from this limitation but employs the understanding gained by geophysical analysis and estimates the density distribution within appropriate constrains imposed by it.

The rest of the paper is organized as follows. We first discuss the concept of the density map. Then, we discuss the density estimation method that leverages the shape model and the spherical harmonic coefficients estimated from OD. These spherical harmonic coefficients are compared with those of the homogeneous body, and their behavior is studied. Lastly, we use Castalia and 1999 KW4 to test hypotheses on how the density is distributed. The results show that the density estimation technique approximates the spherical harmonic coefficients well and the surface gravity field is accurately mapped.

2. Density maps

If the density distribution is known a priori, the polyhedral gravity field can be used to compute the gravitation around the body. However, the inversion process of estimating the density distribution from a given gravity field is not trivial. This fact can be illustrated in a simple example where there are multiple concentric spheres and shells of the same mass. In Fig. 1, as long as each sphere and shell have the same mass, the gravitation sensed by the spacecraft is identical ($\vec{F}_{grav}$), and there are infinitely many other choices of equal mass spheres/shells that satisfy this condition. This simple example illustrates that the inversion process is not unique, and infinitely many solutions of density distribution exist for a given gravity field.

It is worthwhile to pause for a moment and define the terminology used in this paper. The density distribution contains two