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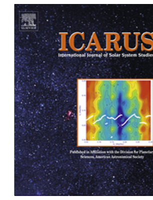

The Poynting–Robertson effect: A critical perspective

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A B S T R A C T

Physics of the Poynting–Robertson (P–R) effect is discussed and compared with the statements published in the past 30 years. Relativistically covariant formulation reveals the essence of the P–R effect and points out to nonphysical explanations in scientific papers and monographs. Although the final equation of motion

$$m\, d\vec{v}/dt = (SA'\overline{Q}'_{pr}/c)[(1 - \vec{v} \cdot \vec{e}/c)\vec{e} - \vec{v}/c]$$

has been usually correctly presented and used, its derivation and explanation of its essence is frequently incorrect.

The difference between the effects of solar electromagnetic and corpuscular (solar wind) radiation is stressed. The force acting on the particle due to the solar wind (the simple case of radial solar wind velocity is considered) is

$$\vec{F}_{sw} = F_{sw}[(1 - \vec{v} \cdot \vec{e}/v_{sw})\vec{e} - x'\vec{v}/v_{sw}],$$

where F_{sw} is the force on the stationary particle, v_{sw} is the heliocentric solar-wind speed, and, the value of x' depends on material properties of the particle ($1 < x' < 3$).

We present secular orbital evolution of dust particle under the action of the P–R effect. Initial conditions are included. Time of spiralling of the particle into the Sun is analytically calculated. Secular evolutions of perihelion and aphelion distances are investigated.

1. Introduction

The force associated with the action of electromagnetic radiation on a moving spherical body (dust particle) is called the Poynting–Robertson (P–R) effect. It is used in astrophysical modelling of orbital evolution of dust grains for many decades. The P–R effect holds when material of the particle is distributed in a spherically symmetric way. It is assumed that the spherical particle can be used as an approximation to real, arbitrarily shaped, particle.

It has elapsed 30 years since the time of Dohnanyi's statement "Much confusion in the literature has existed on this subject and to some extent still does, unfortunately" (Dohnanyi, 1978, p. 563). Conventional idea is that the confusion has been removed in a paper by Burns et al. (1979). In reality, the paper presents derivations and explanations which are in some places inconsistent with Newton's laws, Maxwell's equations, Mie's scattering and relativity theory. However, the paper by Burns et al. (1979) is conventionally considered to be the most relevant paper on the P–R effect (often the only reference on the effect in current papers is that to Burns et al., 1979, see, e.g., Mann, 2010). Incorrect derivations and explanations are often presented also in textbooks (e.g., de Pater and Lissauer, 2010, pp. 45–48). Such derivations and explanations do not enable any real progress in treating evolution of cosmic dust particles. Thus, we can say that the situation has not changed since the time of Dohnanyi's statement. Many incorrect explanations of the P–R effect have been offered during the past 30 years. This avalanche propagation of non-physical statements is strong motivation to write a paper presenting comments on various incorrect derivations and explanations. This will lead to a deeper and more complete understanding of the effect, including similar effect generated by stellar winds. As a consequence, it can improve our understanding of evolution of dust around stars. This paper presents statements on the P–R effect existing in the literature. The paper shows physically incorrect points of the statements and puts them into a correct way (see Section 5). Our paper presents the first fully correct derivation and explanation of the P–R effect published in astronomical or astrophysical journal. The paper should ensure that only correct explanations will be published in future.

To be sure that the physical results are correct, one has to consider relativistically covariant equation of motion. Really, even the

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simplest case of Poynting (1904) and Robertson (1937) is correctly understood only on the basis of relativistic approach and not using classical (nonrelativistic) physics. Moreover, the dimensionless efficiency factors $\overline{Q}'_{pr}$ and $\overline{Q}'_{ext}$ are obtained from Lorenz–Mie's solution of Maxwell's equations and it cannot be said that Maxwell's equations belong to classical (nonrelativistic) physics.

The paper discusses also secular evolution of orbital elements. Initial conditions are included. An analytical expression for time of spiralling of the dust particle into the central star is presented. We also discuss secular evolution of pericentric and apocentric distances of the particle's orbit.

Sections 2–4 present the most relevant results on the physics of the P–R effect. Section 5 discusses various incorrect explanations of the P–R effect, published during the past 30 years. The scientists should concentrate on Section 5.1, where comments on the most frequently cited paper (conventionally considered to be the most relevant paper on the P–R effect) by Burns et al. (1979) are given. The case of the aberration of light is discussed in detail in Section 5.2. Section 6 offers simple correct explanations of the P–R effect. Finally, Section 7 summarizes the relevant equations for the secular evolution of orbital elements of a spherical body under the action of central star's gravity and electromagnetic radiation.

2. Covariant formulations

Relativistically covariant formulations are useful in understanding of the interaction between a particle and an incident electromagnetic radiation. Spherically symmetric mass distribution within the particle is tacitly assumed.

The force associated with the action of electromagnetic radiation on the spherical particle is known as the Poynting–Robertson effect. In the reference frame of the particle spherical symmetry causes that emission component of the radiation force, due to thermal emission, is zero. In other words, thermal emission alone (due the assumed symmetry) does not produce a net force in the reference frame of the particle. The conditions for the incoming and outgoing radiation are then (Appendix A):

$$\frac{dp^{\mu}_{in}}{d\tau} = \frac{w^2 S A' \overline{Q}'_{ext}}{c} b^{\mu},$$
$$\frac{dp^{\mu}_{out}}{d\tau} = \frac{w^2 S A' \overline{Q}'_{ext}}{c} \left[\overline{Q}' \frac{u^{\mu}}{c} + \left(1 - \overline{Q}'\right) b^{\mu} \right],$$
$$\overline{Q}' \equiv \frac{\overline{Q}'_{pr}}{\overline{Q}'_{ext}}, \qquad (1)$$

where the left-hand sides contain four-momenta $p^{\mu}_{in} = (E_{in}/c; \vec{p}_{in})$ and $p^{\mu}_{out} = (E_{out}/c; \vec{p}_{out})$, τ is the time measured in the proper reference frame of the particle, c is the speed of light in vacuum, S is the flux density of radiation energy (energy flow through unit area perpendicular to the ray per unit time), A' is geometrical cross-section of the spherical particle, $\overline{Q}'_{ext}$ is dimensionless extinction efficiency factor given by optical properties of the particle and averaged over the stellar spectrum, $\overline{Q}'_{pr}$ is dimensionless efficiency factor for radiation pressure averaged over stellar spectrum and calculated for radial direction (as for the dimensionless factors of effectivity of radiation pressure and extinction, see Lorenz, 1890; Mie, 1908, or, e.g., Klačka and Kocifaj, 2007, or, Section 4.5 in Bohren and Huffman, 1983). u^{μ} (four-velocity), b^{μ} and the quantity w are given by the following equations:

$$u^{\mu} = (\gamma c; \gamma \vec{v}),$$
$$b^{\mu} = \left(\frac{1}{w}; \frac{\vec{e}}{w}\right),$$
$$w = \gamma \left(1 - \frac{\vec{v} \cdot \vec{e}}{c}\right), \qquad (2)$$

where $\vec{v}$ is particle's velocity in the reference frame of the star, $\vec{e}$ is unit position vector of the particle with respect to the star and γ is the Lorentz factor. Eq. (1) differs from the approach presented in the literature, including Poynting and Robertson, by using the extinction cross-section $\overline{C}'_{ext} = A' \overline{Q}'_{ext}$ as the total cross-section for all photons interacting with the particle.

Equations for $dp^{\mu}_{out}/d\tau$ and $dp^{\mu}_{in}/d\tau$ yield the following covariant relation between the outgoing and the incoming radiation:

$$\frac{dp^{\mu}_{out}}{d\tau} = \left(1 - \overline{Q}'\right) \frac{dp^{\mu}_{in}}{d\tau} + \frac{w^2 S A' \overline{Q}'_{ext}}{c} \overline{Q}' \frac{u^{\mu}}{c},$$
$$\overline{Q}' \equiv \frac{\overline{Q}'_{pr}}{\overline{Q}'_{ext}}. \qquad (3)$$

2.1. Incoming radiation and particle's acceleration

The action of the incoming radiation influences particle's motion according to the following equation of motion:

$$\frac{dp^{\mu}}{d\tau} = \frac{dp^{\mu}_{in}}{d\tau}, \qquad (4)$$

where the right-hand side is given by Eq. (1) and

$$p^{\mu} = m u^{\mu} \qquad (5)$$

is a four-momentum of the particle. Inserting Eqs. (1) and (5) into Eq. (4), we obtain

$$m \frac{du^{\mu}}{d\tau} + \frac{dm}{d\tau} u^{\mu} = \frac{w^2 S A' \overline{Q}'_{ext}}{c} b^{\mu}. \qquad (6)$$

Multiplication of Eq. (6) by u_{μ} yields, on the basis of $u_{\mu} du^{\mu}/d\tau = 0,\ u_{\mu} u^{\mu} = c^2,\ u_{\mu} b^{\mu} = c$:

$$\left(\frac{dm}{d\tau}\right)_{in} = \frac{w^2 S A' \overline{Q}'_{ext}}{c^2}. \qquad (7)$$

Inserting Eq. (7) into Eq. (6), we obtain four-acceleration

$$\left(\frac{du^{\mu}}{d\tau}\right)_{in} = \frac{w^2 S A' \overline{Q}'_{ext}}{mc} \left(b^{\mu} - \frac{u^{\mu}}{c}\right). \qquad (8)$$

The result represented by Eq. (7) can be understood in the sense that $dE_{in}/d\tau = w^2 S A' \overline{Q}'_{ext}$ corresponds to the energy of the incoming radiation interacting with the particle $[dE_{in}/d\tau = c^2 (dm/d\tau)_{in}]$. The real system consists of radiation and the particle and the system cannot be divided into the individual components. Thus, the result presented by Eq. (7) does not correspond to the real change of the particle mass, but only to a virtual change. No direct real experiment can verify the validity of Eq. (7). Indirect evidence of Eq. (7) is the validity of the result $0 \leqslant \overline{Q}'_{pr}/\overline{Q}'_{ext} \leqslant 2$ proven in Appendix D (see also Klačka, 2011a) and experimental confirmation of Maxwell's equations and relativity theory.

It may seem surprising that $(dm/d\tau)_{in}$ depends on the particle velocity and $A' \overline{Q}'_{ext}$, only. E.g., perfectly reflecting and absorbing spherical surfaces, within geometrical optics approximation, yield the same result for $(dm/d\tau)_{in}$.

2.2. Outgoing radiation and particle's acceleration

The action of the outgoing radiation influences the motion of the particle according to the following equation of motion:

$$\frac{dp^{\mu}}{d\tau} = -\frac{dp^{\mu}_{out}}{d\tau}, \qquad (9)$$

where the right-hand side is given by Eq. (1). Inserting Eqs. (1) and (5) into Eq. (9), we obtain