Icarus 231 (2014) 34–50

Contents lists available at ScienceDirect

Icarus

journal homepage: www.elsevier.com/locate/icarus

Non-radial oscillations in rotating giant planets with solid cores: Application to Saturn and its rings

Jim Fuller *, Dong Lai, Natalia I. Storch

Center for Space Research, Department of Astronomy, Cornell University, Ithaca, NY 14853, USA

ARTICLE INFO

Article history:
Received 30 August 2013
Revised 12 November 2013
Accepted 15 November 2013
Available online 24 November 2013

Keywords:
Interiors
Jovian planets
Saturn, interior
Saturn, rings

ABSTRACT

Recent observations have revealed evidence for the global acoustic oscillations of Jupiter and Saturn. Such oscillations can potentially provide a new window into the interior structure of giant planets. Motivated by these observations, we study the non-radial oscillation modes of giant planets containing a solid core. Our calculations include the elastic response of the core and consider a wide range of possible values of the core shear modulus. While the elasticity of the core only slightly changes the frequencies of acoustic modes (including the f-modes), which reside mostly in the fluid envelope, it adds two new classes of shear modes that are largely confined to the core. We also calculate the effects of the Coriolis force on the planetary oscillation modes. In addition to changing the mode frequencies, the Coriolis force can cause the shear modes to mix with the f-modes. Such mixing occurs when the frequencies of the shear mode and the f-mode are close to each other, and results in "mixed modes" that have similarly large surface displacements and gravitational potential perturbations, but are slightly split in frequency. We discuss our results in light of the recent work by Hedman and Nicholson (Hedman, M., Nicholson, P. [2013]. arXiv:1304.3735), which revealed the presence of density waves in Saturn's C-ring that appear to be excited by the gravitational perturbations associated with the f-mode oscillations within Saturn. We find that the fine splitting in wave frequencies observed in the rings can in principle be explained by the rotation-induced mixing between core shear modes and f-modes, possibly indicating the presence of a solid core within Saturn. However, in our current calculations, which assume rigid-body rotation and include only first-order rotational effects, significant fine-tuning in the planetary model parameters is needed in order to achieve these mode mixings and to explain the observed fine frequency splitting. We briefly discuss other effects that may modify the f-modes and facilitate mode mixing.

1. Introduction

Despite enormous advances in precision measurements of various global quantities of giant planets in our Solar System (mass, radius, rotation rate, gravitational moments, oblateness, etc.), our knowledge of the interior structure of these planets is rather limited. One uncertainty is the size of the central core, with estimates in the range of $\sim(0–10)M_\oplus$ (Guillot, 2005) and $\sim(14–18)M_\oplus$ (Militzer et al., 2008) for Jupiter, and $\sim(9–22)M_\oplus$ (Guillot, 2005) for Saturn. Another uncertainty concerns the mixing and possible stratification of heavy elements in the core and fluid envelope (Stevenson, 1985; Leconte and Chabrier, 2012).

Global seismology is a promising technique for probing the internal structures of stars and planets. Indeed, the internal structure of the Earth, Moon, Sun, and numerous types of stars has been constrained primarily via measurements of global oscillations (see Unno et al. (1989) for stellar oscillations, (Dahlen and Tromp, 1998), hereafter DT98, for a comprehensive description of the techniques of Earth seismology, (Chaplin and Miglio, 2013) a review of recent developments in asteroseismology, and (Lognonne and Mosser, 1993 and Stein and Wysession, 2003) for results in terrestrial seismology). Unfortunately, direct detection of global oscillations in giant planets is extremely difficult because the oscillations produce negligible luminosity perturbations and have small surface displacements (radial surface displacements are likely on the order centimeters). Recently, Gaulme et al. (2011) reported the detection of acoustic modes (p-modes) in the radial velocity data of Jupiter, but the quality of the data was insufficient to provide new constraints on Jupiter's interior model.

Saturn's ring system offers a unique opportunity to perform planetary seismology, because even mild gravitational perturbations associated with the planet's oscillation modes can generate density waves that propagate through the rings. Marley and Porco (1993) investigated this idea in detail, arguing that some of the unexplained wave features (Rosen et al., 1991) in Saturn's C and D-rings were produced at Lindblad resonances with the gravitational perturbations associated with Saturn's oscillation modes.

* Corresponding author.
E-mail address: derg@astro.cornell.edu (J. Fuller).

0019-1035/$ - see front matter
http://dx.doi.org/10.1016/j.icarus.2013.11.022

However, the existing *Voyager* data was insufficient to measure the properties of the waves, and so their seismic utility was limited.

Recently, Hedman and Nicholson (2013) (hereafter HN13) used *Cassini* occultation data (see Colwell et al., 2009; Baillie et al., 2011) to measure the radial location of Lindblad resonance (r_L), azimuthal pattern number (m), and angular pattern frequency (Ω_p) of several waves in Saturn's C-ring. We retabulate the results of HN13 in Table 1. HN13 demonstrated that these waves cannot be excited by resonances with any of Saturn's (known or unknown) satellites, but are compatible with being excited by low degree prograde sectoral ($l = |m| = 2, 3, 4$) fundamental oscillation modes (f-modes) of Saturn (as predicted by Marley (1991) and Marley and Porco (1993)). Throughout this paper, we adopt the convention that perturbations have the form $e^{im\phi+i\sigma t}$, with the inertial frame mode frequency $\sigma = |m|\Omega_p > 0$.

Intriguingly, HN13 found what appeared to be a "fine splitting" in the mode frequencies: Instead of one $m = -2$ wave excited by Saturn's $l = 2, m = -2$ (prograde) f-mode, there are *two* discrete waves with a frequency difference of about 4%; instead of one $m = -3$ wave excited by the $l = 3, m = -3$ f-mode, there are *three* waves with a frequency difference of $(0.1\text{–}0.3)\%$. No fine splitting was observed for the $m = -4$ wave, and no waves with $m < -4$ were observed.

The origin of these fine splittings is puzzling. Saturn rotates rapidly (with a spin period of 10.6 hours), which splits the f-mode of a given degree l into multiples with azimuthal order $m = -l, -l+1, \cdot, l-1, l$. For a given m, there are many f-modes that correspond to different degrees l in the norotating configuration, but these modes differ in frequency by order unity (for small l), much larger than the observed splitting. Rotation can also introduce other Coriolis-force-supported modes (inertial modes and Rossby modes), but these modes all have frequencies (in the rotating frame) less than $2\Omega_s \simeq 1630°$/day (where Ω_s is Saturn's rotation rate), which are smaller than the f-mode frequencies. We discuss some other possible effects in Section 6.

In this paper, we explore the properties of oscillation modes of giant planets that contain a solid core. We have two goals. (1) Previous calculations of the oscillation modes of giant planets have been restricted to pure fluid models, with or without a dense core (e.g., Vorontsov and Zharkov, 1981; Vorontsov, 1981, 1984; Marley, 1991; Wu, 2005; Le Bihan and Burrows, 2012). Although the global oscillations of the solid Earth are well studied (DT98) and there have been some studies on the effects of elasticity of the solid cores/crusts in white dwarfs and neutron stars (e.g., Hansen and Van Horn, 1979; McDermott et al., 1988; Montgomery and Winget, 1999), to our knowledge, no previous works have investigated the elastic response of a solid core in giant planets. The elasticity of a solid core adds entire new classes of modes that have previously been ignored and can also modify the properties of fluid modes (such as f-modes), which may have observable signatures. (2) We examine the possibility of rotational mixing between elastic core modes and envelope f-modes. While the influences of rotation on the mode frequencies are well studied in stars (Unno et al., 1989) and have been included in previous works on giant planetary oscillations (e.g., Vorontsov, 1981; Marley, 1991), the possibility of rotation-induced mode mixing with elastic core modes has not been investigated. We show that such mode mixing can in principle lead to the appearance of multiple oscillation modes having very similar frequencies and characteristics. Nevertheless, as we show in this paper, significant fine tuning of the planetary model parameters is needed to produce the observed fine splitting of the waves in Saturn's rings.

Our paper is organized as follows. In Section 2, we generate simple giant planet models that will serve as the basis of our oscillation mode calculations. Section 3 describes the characteristics of oscillations in non-rotating planets, while Section 4 investigates mode mixing in rotating planets. In Section 5, we calculate the effects of oscillation modes on Saturn's rings, and we compare our results to the observations of HN13. In Section 6, we summarize our results and discuss other effects (such as differential rotation and magnetic fields) that may modify the oscillation modes and influence mode mixing.

Table 1
Properties of the waves in Saturn's C-ring measured by HN13. The waves have the form $e^{im\phi+i\sigma t}$, with the wave frequency $\sigma = |m|\Omega_p$. Resonant locations are measured from Saturn's center, and are taken from Baillie et al. (2011). The value of $|\delta\tau|$ is the approximate maximum semi-amplitude of the optical depth variation associated with each wave.

Wave	Resonant location	m	Ω_p (deg/day)	$\lvert\delta\tau\rvert$
W80.98	80,988 km	−4	1660.3	0.09
W82.00	82,010 km	−3	1736.6	0.07
W82.06	82,061 km	−3	1735.0	0.21
W82.21	82,209 km	−3	1730.3	0.15
W84.64	84,644 km	−2	1860.8	0.09
W87.19	87,189 km	−2	1779.5	0.14

2. Planetary model

Since our goal is to understand the effect of core elasticity on the oscillations of giant planets and to explore the possibility of rotation-induced mode mixing, we will not use sophisticated giant planet models with "realistic" equation of state (e.g., Guillot, 2005) in this paper. Instead, we will consider simple planet models composed of a one-component solid core surrounded by a neutrally stratified fluid envelope characterized by a $n = 1$ ($\Gamma = 2$) polytropic equation of state. These models allow us to capture the basic properties of giant planets without getting bogged down in uncertain details (e.g., helium rain out, liquid–metallic hydrogen phase transitions, core size and composition, etc.).

To generate our planet models, we first construct a polytropic model of index $n = 1$ (so that the pressure is related to density as $P \propto \rho^2$). We then add a solid core by choosing a core radius, R_c, a dimensionless density enhancement D, and constant shear modulus μ (the shear modulus of the fluid envelope is zero). The density of material in the core is calculated by multiplying the density of material with $r < R_c$ in the original polytropic model by D. We then normalize the density profile so that the total mass/radius equal the mass/radius of Saturn. With this density profile, we compute the gravitational acceleration via $g = GM(r)/r^2$, where $M(r) = \int_0^r 4\pi r^2 \rho dr$. We then assume the planet is neutrally stratified at all radii such that the Brunt–Vaisala frequency $N^2 = 0$. The pressure P is obtained by integrating the hydrostatic equilibrium equation $dP/dr = -\rho g$, and the bulk modulus K is given by

$$K = -\rho g \left(\frac{d\ln\rho}{dr}\right)^{-1}. \tag{1}$$

The bulk modulus is related to the pressure P via $K = \Gamma_1 P$, with $\Gamma_1 = d\ln P/d\ln\rho$.

For the purposes of calculating adiabatic acoustic-elastic pulsations in non-rotating spherically symmetric planets, a planetary model is completely described by three quantities as a function of radius: the density ρ, adiabatic bulk modulus K, and the shear modulus μ (see elastic oscillation equations in Section 3). Thus, our models have four free parameters: the density profile index[1] n, the radius of the solid core R_c (or more precisely, the ratio of R_c to the planet radius R), the core-envelope density jump D, and the

[1] Note that n is the polytropic index (as in $P \propto \rho^{1+1/n}$) of the planet without a core; when a core is added, the pressure and density in the fluid envelope no longer satisfies the polytropic relation.