Icarus 231 (2014) 206–220

Contents lists available at ScienceDirect

Icarus

journal homepage: www.elsevier.com/locate/icarus

Effect of core–mantle and tidal torques on Mercury's spin axis orientation

Stanton J. Peale [a,*], Jean-Luc Margot [b,c], Steven A. Hauck II [d], Sean C. Solomon [e,f]

[a] Department of Physics, University of California, Santa Barbara, CA 93106, United States
[b] Department of Earth, Planetary, and Space Sciences, University of California, Los Angeles, CA 93005, United States
[c] Department of Physics and Astronomy, University of California, Los Angeles, CA 90095, United States
[d] Department of Earth, Environmental, and Planetary Sciences, Case Western Reserve University, Cleveland, OH 44106, United States
[e] Department of Terrestrial Magnetism, Carnegie Institution of Washington, Washington, DC 20015, United States
[f] Lamont-Doherty Earth Observatory, Columbia University, Palisades, NY 10964, United States

ARTICLE INFO

Article history:
Received 14 August 2013
Revised 5 December 2013
Accepted 6 December 2013
Available online 17 December 2013

Keywords:
Rotational dynamics
Mercury, interior
Planetary dynamics
Tides, solid body

ABSTRACT

The rotational evolution of Mercury's mantle plus crust and its core under conservative and dissipative torques is important for understanding the planet's spin state. Dissipation results from tidal torques and viscous, magnetic, and topographic torques contributed by interactions between the liquid core and solid mantle. For a spherically symmetric core–mantle boundary (CMB), the system goes to an equilibrium state wherein the spin axes of the mantle and core are fixed in the frame precessing with the orbit, and in which the mantle and core are differentially rotating. This equilibrium exhibits a mantle spin axis that is offset from the Cassini state by larger amounts for weaker core–mantle coupling for all three dissipative core–mantle coupling mechanisms, and the spin axis of the core is separated farther from that of the mantle, leading to larger differential rotation. Relatively strong core–mantle coupling is necessary to bring the mantle spin axis to a position within the uncertainty in its observed position, which is close to the Cassini state defined for a completely solid Mercury. Strong core–mantle coupling means that Mercury's response is closer to that of a solid planet. Measured or inferred values of parameters in all three core–mantle coupling mechanisms for a spherically symmetric CMB appear not to accomplish this requirement. For a hydrostatic ellipsoidal CMB, pressure coupling dominates the dissipative effects on the mantle and core positions, and dissipation with pressure coupling brings the mantle spin solidly to the Cassini state. The core spin goes to a position displaced from that of the mantle by about 3.55 arcmin nearly in the plane containing the Cassini state. The core spin lags the precessing plane containing the Cassini state by an increasing angle as the core viscosity is increased. With the maximum viscosity considered of $\nu \sim 15.0\,\mathrm{cm}^2/\mathrm{s}$ if the coupling is by the circulation through an Ekman boundary layer or $\nu \sim 8.75 \times 10^5\,\mathrm{cm}^2/\mathrm{s}$ for purely viscous coupling, the core spin lags the precessing Cassini plane by 23 arcsec, whereas the mantle spin lags by only 0.055 arcsec. Larger, non-hydrostatic values of the CMB ellipticity also result in the mantle spin at the Cassini state, but the core spin is moved closer to the mantle spin. Current measurement uncertainties preclude using the mantle offset to constrain the internal core viscosity.

1. Introduction

Mercury is in a stable spin–orbit resonance in which the rotational angular velocity is precisely 1.5 times the mean orbital motion (Pettengill and Dyce, 1965; Colombo and Shapiro, 1966). This rotation state is a natural outcome of tidal evolution (Goldreich and Peale, 1966; Correia and Laskar, 2004, 2009). In addition, the same tidal evolution brings Mercury to Cassini state 1, wherein Mercury's spin axis remains coplanar with the orbit normal and Laplace plane normal as the spin vector and orbit normal precess around the latter with a ~300,000 yr period (Colombo, 1966; Peale, 1969, 1974). That Mercury is very close to this state has been verified with radar observations, which give an obliquity of 2.04 ± 0.08 arcmin (Margot et al., 2007, 2012). The most recent observations show that the best-fit solution is offset from the Cassini state by a few arcseconds, but the uncertainty at one standard deviation includes the Cassini state.

This paper is an investigation of the possible displacement of the spin axis from the Cassini state from dissipative processes and the consequences of pressure coupling. In Section 2 we develop the equations for the rotational motion of both the core and the mantle plus crust from conservative and dissipative torques. The latter include the tidal torque and the torques due to viscous, mag-

---

[*] Corresponding author.
E-mail address: peale@physics.ucsb.edu (S.J. Peale).

0019-1035/$ - see front matter © 2013 Elsevier Inc. All rights reserved.
http://dx.doi.org/10.1016/j.icarus.2013.12.007

netic, and topographical coupling between the core and mantle for a spherically symmetric core–mantle boundary (CMB). Gravitational and rotational distortions of the CMB lead to pressure torques that dominate all the dissipative torques. Results are given in Section 3, where we show that the tidal offset of the mantle spin axis from the Cassini state is immeasurably small, but the offset due to the core–mantle interactions can be quite large, and weaker core–mantle coupling leads to larger offsets. The core–mantle dissipative coupling must be relatively strong to bring the mantle spin-axis to within the uncertainty of its observed location. The failure of viscous, magnetic and topographic mechanisms, which dominate the tidal mechanism, to bring the spin axis near its observed position for measured or likely values of the parameters is compensated by the pressure coupling between the core and mantle for both hydrostatic and non-hydrostatic ellipsoidal CMB, which we examine in Section 3.5.

We maintain the current orbital configurations throughout the calculations even though the dissipative time scales are long enough for significant changes to occur. This assumption is justified because the spin axis will follow the Cassini state as the latter's position changes during the slow changes in the Solar System configuration because of adiabatic invariance of the solid angle swept out by the spin axis as it precesses around the Cassini state. The spin axis remains within 1 arcsec of the Cassini state position through both long-period and short-period changes in the state position (Peale, 2006). We are interested only in the final equilibrium positions of the core and mantle spins in the current orbit frame of reference, and these positions will be the same if the evolution takes place with the current, fixed orbital and Solar System parameters or if these parameters are allowed to evolve during the evolution to the current state.

2. Equations of variation

The coordinate systems and angles for the equations that govern the rotational motion of Mercury are shown in Fig. 1, where X', Y', Z' are quasi-inertial axes with the $X'Y'$ plane being the Laplace plane on which Mercury's orbit precesses at nearly a constant inclination I and nearly constant angular velocity $\boldsymbol{\mu}$. The XYZ orbit system has the X axis along the ascending node of the orbit plane on the Laplace plane, and the XY plane is the orbit plane. The xyz system is fixed in the body, with z along the spin axis and x along the axis of minimum moment of inertia in the equator plane. The Euler angles orienting the xyz system relative to the XYZ system are Ω, i, ψ, where Ω is the longitude of the ascending node of the equator plane on the XY orbit plane measured from the X axis, i is the inclination of the equator plane to the orbit plane, and ψ is the angle from the ascending node of the equator on the orbit plane to the x axis of minimum moment of inertia. The three Euler angles will have subscripts m or f to designate mantle or fluid core, respectively. Angle I is the inclination of the orbit plane to the Laplace plane, Ω_o is the longitude of the ascending node of the orbit plane on the Laplace plane, ω is the argument of perihelion, f is the true anomaly of the Sun, and r is the distance from Mercury to the Sun.

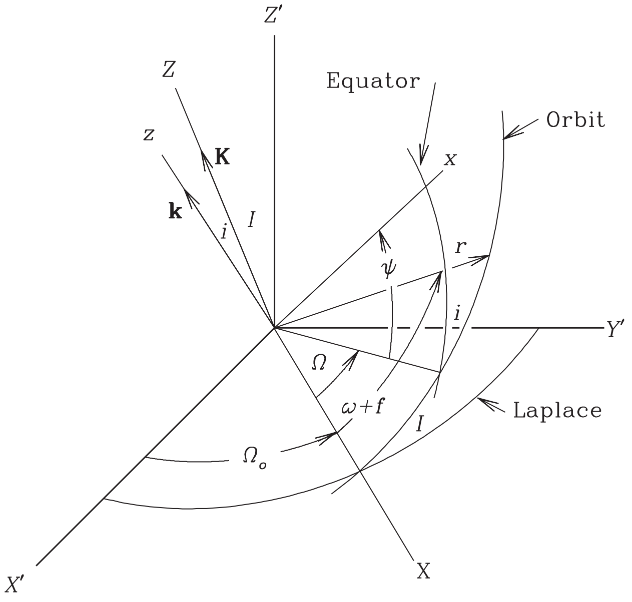

Fig. 1. Coordinate systems and relevant angles. The angles orienting mantle or core relative to the XYZ orbit system will have subscripts m or c, respectively.

We assume principal axis rotation throughout. The angular momentum of the mantle plus crust is $\mathbf{L_m} = C_m\dot\psi_m\mathbf{k}_m = C_m\dot{\boldsymbol{\psi}}_m$, where C_m is the moment of inertia of the mantle plus crust about the spin axis, $\dot\psi_m$ is the angular velocity of the mantle, and $\mathbf{k}_m = \sin i_m \sin\Omega_m\mathbf{I} - \sin i_m\cos\Omega_m\mathbf{J} + \cos i_m\mathbf{K}$ is a unit vector along the spin axis. $\mathbf{I}, \mathbf{J}, \mathbf{K}$ are unit vectors along the X, Y, Z axes, respectively. With $d\mathbf{L}_m/dt = C_m(d\dot\psi_m/dt)\mathbf{k}_m + C_m\dot\psi_m(d\mathbf{k}_m/dt)$, we can write

$$\frac{1}{C_m}\frac{dL_{mX}}{dt}=\frac{d\dot\psi_m}{dt}\sin i_m\sin\Omega_m+\dot\psi_m\left[\cos i_m\sin\Omega_m\frac{di_m}{dt}+\sin i_m\cos\Omega_m\frac{d\Omega_m}{dt}\right]$$
$$\frac{1}{C_m}\frac{dL_{mY}}{dt}=-\frac{d\dot\psi_m}{dt}\sin i_m\cos\Omega_m+\dot\psi_m\left[-\cos i_m\cos\Omega_m\frac{di_m}{dt}+\sin i_m\sin\Omega_m\frac{d\Omega_m}{dt}\right]$$
$$\frac{1}{C_m}\frac{dL_{mZ}}{dt}=\frac{d\dot\psi_m}{dt}\cos i_m-\dot\psi_m\sin i_m\frac{di_m}{dt} \tag{1}$$

for the variations of the three components of angular momentum relative to the orbit system of coordinates, which system is readily observable.

The total torque on Mercury's mantle plus crust $\langle\mathbf{T_m}\rangle = \langle\mathbf{T}_{body}\rangle + \langle\mathbf{T}_{tide}\rangle + \langle\mathbf{T}_{f-m}\rangle$, is the sum of the conservative gravitational torque, the tidal torque, and the torque from the core–mantle interaction. The latter torque has four contributions, $\langle\mathbf{T}_{viscous}\rangle$, $\langle\mathbf{T}_{magnetic}\rangle$, $\langle\mathbf{T}_{topographic}\rangle$, and $\langle\mathbf{T}_{pressure}\rangle$, for viscous, magnetic, topographic, and pressure coupling, respectively. The angled brackets indicate that these torques are averaged over an orbit period; the core–mantle torques do not involve the orbital elements, so they are intrinsically averaged. We desire the variation of $\mathbf{L_m}$ relative to the precessing orbit system, where the variation relative to inertial space is given by the total torque. We therefore write $d\mathbf{L_m}/dt = \langle\mathbf{T_m}\rangle - \boldsymbol{\mu}\times\mathbf{L_m}$, where $\boldsymbol{\mu}$ is the angular velocity of the orbit precession. If we write $\mathbf{N_m} = \langle\mathbf{T_m}\rangle/C_m - \boldsymbol{\mu}\times\dot{\boldsymbol{\psi}}_m$, we can equate each component of $(1/C_m)d\mathbf{L_m}/dt$ in Eq. (1) to the corresponding component of $\mathbf{N_m}$ and solve the resulting set for $d\dot\psi_m/dt$, di_m/dt, and $d\Omega_m/dt$ to follow the motion of Mercury's mantle under conservative and dissipative torques. We find

$$\frac{d\dot\psi_m}{dt}=\sin i_m(N_{mX}\sin\Omega_m-N_{mY}\cos\Omega_m)+N_{mZ}\cos i_m,$$
$$\frac{di_m}{dt}=-\frac{1}{\dot\psi_m}[\cos i_m(-N_{mX}\sin\Omega_m+N_{mY}\cos\Omega_m)+N_{mZ}\sin i_m],$$
$$\frac{d\Omega_m}{dt}=\frac{1}{\dot\psi_m\sin i_m}(N_{mX}\cos\Omega_m+N_{mY}\sin\Omega_m). \tag{2}$$

We change variables to $p_m = \sin i_m\sin\Omega_m$ and $q_m = \sin i_m\cos\Omega_m$ to eliminate the $\sin i_m$ singularity in the third of Eq. (2). Differentiating these variables with respect to time, substituting the expressions for the time derivatives from Eq. (2), and expressing the circular functions in terms of p_m and q_m yields

$$\frac{d\dot\psi_m}{dt}=p_mN_{mX}-q_mN_{mY}+\sqrt{1-p_m^2-q_m^2}N_{mZ}$$
$$\frac{dp_m}{dt}=\frac{1}{\dot\psi_m}\left[(1-p_m^2)N_{mX}+p_mq_mN_{mY}-p_m\sqrt{1-p_m^2-q_m^2}N_{mZ}\right]$$
$$\frac{dq_m}{dt}=-\frac{1}{\dot\psi_m}\left[p_mq_mN_{mX}+(1-q_m^2)N_{mY}+q_m\sqrt{1-p_m^2-q_m^2}N_{mZ}\right] \tag{3}$$