Icarus 229 (2014) 418–422

Contents lists available at ScienceDirect

Icarus

journal homepage: www.elsevier.com/locate/icarus

Note

Tidal end states of binary asteroid systems with a nonspherical component

Patrick A. Taylor[a,*], Jean-Luc Margot[b]

[a] Arecibo Observatory, HC 3 Box 53995, Arecibo, PR 00612, USA
[b] Departments of Earth, Planetary, & Space Sciences and Physics & Astronomy, University of California, Los Angeles, 595 Charles Young Drive East, Los Angeles, CA 90095, USA

ARTICLE INFO

Article history:
Received 22 August 2013
Revised 4 November 2013
Accepted 5 November 2013
Available online 15 November 2013

Keywords:
Asteroids
Satellites of asteroids
Tides, solid body
Asteroids, dynamics
Asteroids, rotation

ABSTRACT

We derive the locations of the fully synchronous end states of tidal evolution for binary asteroid systems having one spherical component and one oblate- or prolate-spheroid component. Departures from a spherical shape, at levels observed among binary asteroids, can result in the lack of a stable tidal end state for particular combinations of the system mass fraction and angular momentum, in which case the binary must collapse to contact. We illustrate our analytical results with near-Earth Asteroids (8567) 1996 HW_1, (66391) 1999 KW_4, and 69230 Hermes.

1. Introduction

Recent studies have examined energy, stability, and orbital relative equilibria in the planar two-body problem for a non-rotating sphere and an arbitrary, rotating ellipsoid (Scheeres, 2007; Bellerose and Scheeres, 2008) and approximately for two arbitrary, rotating ellipsoids (Scheeres, 2009). Here, we examine the special case of a rotating sphere interacting with a rotating oblate or prolate spheroid and provide exact, tractable analytical results for the locations of the fully synchronous end states of tidal evolution. The terms fully synchronous tidal end state and orbital relative equilibrium can be used interchangeably to describe a zero-eccentricity binary system that has ceased tidally evolving because the spin rates of both components have synchronized to the mean motion of the components about the center of mass of the system.

This note is organized as follows. In Section 2, we review fully synchronous tidal end states of a binary system consisting of two spheres. Section 3 extends the discussion to a sphere interacting with an ellipsoid and explores the specific cases of oblate and prolate spheroids with applications to real asteroid systems. Comparisons to previous work in Sections 3 and 4 place this work in context and possible avenues for contact-binary formation are suggested.

2. Fully synchronous orbits with spherical components

The locations of the fully synchronous end states of tidal evolution for binary asteroids with spherical components were discussed by Taylor and Margot (2011) and are summarized here. For components of equal, uniform density ρ with radii R_1 and R_2 and mass ratio $q = M_2/M_1 = (R_2/R_1)^3$ separated by a distance a in their circular mutual orbit, the sum of the orbital and spin angular momentum J upon full synchronization, scaled by $J' = \sqrt{G(M_1+M_2)^3 R_{\rm eff}}$, where $R_{\rm eff}$ is the effective radius of a sphere with the same volume as both components combined, is:

$$\frac{J}{J'} = \frac{q}{(1+q)^{13/6}}\left(\frac{a}{R_1}\right)^{1/2} + \frac{2}{5}\,\frac{1+q^{5/3}}{(1+q)^{7/6}}\left(\frac{a}{R_1}\right)^{-3/2} \quad (1)$$

[cf. Taylor and Margot (2011), Eq. (8)]. The term on the left, proportional to $a^{1/2}$, is the orbital angular momentum of the system revolving with mean motion n, given by Kepler's Third Law, scaled by J'. The term on the right, proportional to $a^{-3/2}$, is the spin angular momentum of the two components, both rotating with spin rate n, scaled by J'. The $1 + q^{5/3}$ term is proportional to the sum of the moments of inertia of the two bodies; removing the $q^{5/3}$ term amounts to ignoring the spin angular momentum of component 2. Depending on the mass ratio and the total angular momentum of the system, Eq. (1) may have zero, one (degenerate), or two solutions (one unstable and one stable), corresponding to the number of fully synchronous orbits supported by the system. The total energy when the system has fully synchronized may be positive

* Corresponding author. Fax: +1 787 878 1861.
E-mail addresses: ptaylor@naic.edu (P.A. Taylor), jlm@astro.ucla.edu (J.L. Margot).

0019-1035/$ - see front matter
http://dx.doi.org/10.1016/j.icarus.2013.11.008

or negative depending on the parameters of the system. One can show that the zero-energy limit always falls within the stability limit that splits the unstable and stable solutions such that all stable, fully synchronous orbits have negative energy, i.e., they are gravitationally bound.

For plotting purposes, we transform from mass ratio q to mass fraction $\nu = M_2/(M_1 + M_2)$ and scale the separation a by $R_1 + R_2$, the contact limit. Fig. 1 shows, for a two-sphere binary system, the locations of the fully synchronous orbits using contours of angular momentum J/J'. Because the components are similar in shape, the diagram is mirror symmetric about $\nu = 0.5$; this will not be the case when one component is nonspherical. Unstable inner synchronous orbits, the solutions below the stability limit in Fig. 1, almost always fall within the contact limit, with the exception of the $J/J' = 0.25$ curve, similar to the angular momentum found in most large main-belt binary systems likely formed by collisions. In systems with $J/J' \sim 0.4$, similar to near-Earth binaries and small main-belt binaries likely formed via spin-up processes, the secondary is formed beyond the inner synchronous orbit and will naturally tidally evolve outward, vertically through the diagram, until reaching the outer synchronous orbit at the intersection with its corresponding angular-momentum contour. Of course, this is a simplistic view because the post-fission dynamical environment of a newly formed binary asteroid is chaotic (Jacobson and Scheeres, 2011), carrying the risk of ejection or re-impact of the secondary or the secondary itself undergoing fission. Once the system has settled, the steady, comparatively quiescent, tidal evolution to the outer synchronous orbit continues as in Fig. 1. An equal-mass binary with $\nu = 0.5$ must have $J/J' > 0.44$ (more exactly, 0.43956) to have a stable tidal end state.

3. Fully synchronous orbits with a nonspherical component

Let component 1 of the binary system be a uniform-density ellipsoid with principal semi-axes $a_0 \geqslant a_1 \geqslant a_2$ such that the equivalent radius of the ellipsoid is $R_1 = (a_0 a_1 a_2)^{1/3}$. For rotation about the shortest principal axis, the ratio of the moment of inertia of the ellipsoid to that of its equivalent-volume sphere with radius R_1 is the nonsphericity parameter (Descamps and Marchis, 2008):

$$\lambda = \frac{1 + \beta^2}{2(\alpha\beta)^{2/3}}, \tag{2}$$

where $\alpha = a_2/a_0, \beta = a_1/a_0$, and $\alpha \leqslant \beta \leq 1$. The nonsphericity parameter is always larger than unity because any departure from a spherical shape requires displacing mass farther from the spin axis and increases the moment of inertia of the body. Component 2 is assumed to remain spherical. To retain orbital relative equilibrium, the sphere must orbit above one of the principal axes of the ellipsoid and the system must rotate about another principal axis of the ellipsoid at a specific rate (Scheeres, 2006) given by:

$$n^2 = \frac{3}{2} G(M_1 + M_2) \int_{r^2 - a_i^2}^{\infty} \frac{\mathrm{d}u}{(a_i^2 + u)\Delta(u)} \tag{3}$$

[cf. Scheeres (2007), Eq. (18)], where $\Delta(u) = \sqrt{(a_0^2 + u)(a_1^2 + u)(a_2^2 + u)}$, a_i is the principal semi-axis that the sphere orbits above, and r is the orbital separation of the bodies (r is the semimajor axis a for the circular orbits considered here). Defining $\bar{a} = a/a_0$ and $u' = u/a_0^2$, the mean motion becomes:

$$n^2 = \frac{G(M_1 + M_2)}{a^3} f(\alpha, \beta, \bar{a}, a_i), \tag{4}$$

introducing f as the dimensionless integral:

$$f(\alpha, \beta, \bar{a}, a_i) = \frac{3}{2}\bar{a}^3 \int_{\bar{a}^2 - \left(\frac{a_i}{a_0}\right)^2}^{\infty} \frac{\mathrm{d}u'}{\left(\left(\frac{a_i}{a_0}\right)^2 + u'\right)\sqrt{(1 + u')(\alpha^2 + u')(\beta^2 + u')}}. \tag{5}$$

For two spheres, Eq. (4) simplifies to Kepler's Third Law as the integral f goes to unity. To apply this condition to an ensemble of systems while accounting for the spins of both components and the orbital mean motion, we use a dimensionless form of the angular momentum that is applicable to binary systems with any absolute size, mass, and separation. Starting from Eq. (1), when component 1 is nonspherical, the effective radius R_1 is by definition $(\alpha\beta)^{1/3} a_0$, the contribution of the (scaled) moments of inertia of the two components to the spin angular momentum increases from $1 + q^{5/3}$ to $\lambda + q^{5/3}$, and the mean motion n includes the additional factor of $f(\alpha, \beta, \bar{a}, a_i)^{1/2}$ compared to the two-sphere case. Upon simplification, the total angular momentum J/J' of a sphere and ellipsoid in a fully synchronous orbit satisfies:

$$\frac{J}{J'} = (\alpha\beta)^{-1/6} \left[\frac{q}{(1+q)^{13/6}} \bar{a}^{1/2} + \frac{2}{5} \frac{1}{(1+q)^{7/6}} \left(\frac{1 + \beta^2}{2} + (\alpha\beta)^{2/3} q^{5/3} \right) \bar{a}^{-3/2} \right] [f(\alpha, \beta, \bar{a}, a_i)]^{1/2}, \tag{6}$$

recalling that $\bar{a} = a/a_0$. In the limit that the nonspherical component approaches a sphere, α, β, and the integral f go to unity and $\bar{a}$ is equivalent to a/R_1, which recovers the two-sphere case of Eq. (1) explored by Taylor and Margot (2011) and shown in Fig. 1.

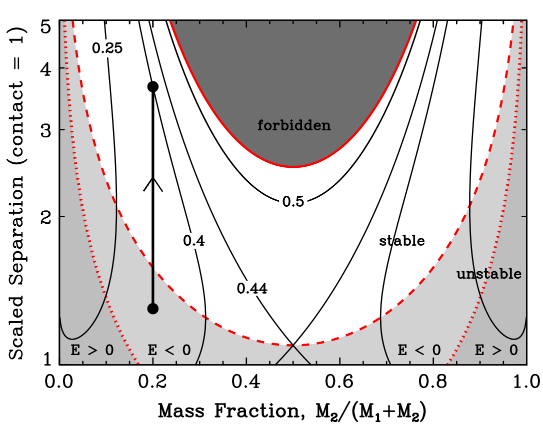

Fig. 1. Component separation a, scaled to the contact limit, for the fully synchronous orbits of a two-sphere binary system with mass fraction ν and angular momentum J/J'. The black curves indicate the inner (when not within contact limit) and outer synchronous orbits for $J/J' = 0.25, 0.4, 0.44$ and 0.5. The red dotted curve is the zero-energy limit; tidal end states above this limit have negative energy ($E < 0$) and must remain bound. The red dashed curve is the stability limit that splits the unstable inner orbits in the gray regions from the stable outer orbits in the white region. The darkest region above the solid red line represents the angular-momentum limit for $J/J' = 0.5$ and is inaccessible to systems with $J/J' \leqslant 0.5$. See Section 3.1 and Taylor and Margot (2011) for details on these limits. A binary system tidally evolves upward along a vertical line at mass fraction ν, away from the gray regions and into the white region, where it reaches the stable outer synchronous orbit at the intersection with its corresponding J/J' contour. A $\nu = 0.2$ ($q = 0.25$) binary system with $J/J' = 0.4$ is shown evolving from a state initially near contact.

3.1. Angular-momentum, stability, and zero-energy limits

Three dynamical limits: the angular-momentum limit, the stability limit, and the zero-energy limit, break up the parameter space of mass fraction and separation, and all three depend on the shape of the nonspherical component of the binary system. The angular-momentum limit follows from Eq. (6) by setting the spin angular momentum (the term proportional to $\bar{a}^{-3/2}$) to zero and rearranging such that the maximum separation of the components $\bar{a}_{\max} = a_{\max}/a_0$ for a given angular momentum J/J' is the numerical solution to: