Journal of Atmospheric and Solar-Terrestrial Physics 172 (2018) 40–52

Contents lists available at ScienceDirect

Journal of Atmospheric and Solar-Terrestrial Physics

journal homepage: www.elsevier.com/locate/jastp

Finite amplitude transverse oscillations of a magnetic rope

Dmitrii Y. Kolotkov [a,*], Giuseppe Nisticò [a,1], George Rowlands [a], Valery M. Nakariakov [a,b]

[a] Centre for Fusion, Space and Astrophysics, Department of Physics, University of Warwick, CV4 7AL, UK
[b] St. Petersburg Branch, Special Astrophysical Observatory, Russian Academy of Sciences, 196140, St. Petersburg, Russia

A R T I C L E I N F O

Keywords:
Quiescent prominences
Filaments
Nonlinear oscillations
Magnetic field

A B S T R A C T

The effects of finite amplitudes on the transverse oscillations of a quiescent prominence represented by a magnetic rope are investigated in terms of the model proposed by Kolotkov et al. (2016). We consider a weakly nonlinear case governed by a quadratic nonlinearity, and also analyse the fully nonlinear equations of motion. We treat the prominence as a massive line current located above the photosphere and interacting with the magnetised dipped environment via the Lorentz force. In this concept the magnetic dip is produced by two external current sources located at the photosphere. Finite amplitude horizontal and vertical oscillations are found to be strongly coupled between each other. The coupling is more efficient for larger amplitudes and smaller attack angles between the direction of the driver and the horizontal axis. Spatial structure of oscillations is represented by Lissajous-like curves with the limit cycle of a hourglass shape, appearing in the resonant case, when the frequency of the vertical mode is twice the horizontal mode frequency. A metastable equilibrium of the prominence is revealed, which is stable for small amplitude displacements, and becomes horizontally unstable, when the amplitude exceeds a threshold value. The maximum oscillation amplitudes are also analytically derived and analysed. Typical oscillation periods are determined by the oscillation amplitude, prominence current, its mass and position above the photosphere, and the parameters of the magnetic dip. The main new effects of the finite amplitude are the coupling of the horizontally and vertically polarised transverse oscillations (i.e. the lack of a simple, elliptically polarised regime) and the presence of metastable equilibria of prominences.

1. Introduction

Solar prominences are the condensations of plasma at temperatures of about 10^4 K (typical for the chromosphere) floating in the much hotter solar corona (with temperatures typically greater than 10^6 K) (see e.g. Parenti, 2014, for a comprehensive review). The main questions related to prominences concern the physical mechanisms involved in their formation and evolution. Indeed, prominences can be generally distinguished in two categories: quiescent prominences, which are observed floating in the low solar corona with time scales ranging from hours to several days before to slowly fade out or dissolve; and erupting prominences, which become unstable in the presence of particular physical conditions. As a consequence of the prominence eruption, a coronal mass ejection (CME) could be formed and expelled from the solar corona. The loss of equilibrium can be caused by various reasons: eruptions can be triggered by a nearby flare (Panesar et al., 2015), or in response to an emerging magnetic flux or variation of the local magnetic helicity (Yeates and Mackay, 2009), or maybe due to the action of MHD waves, as observed for some events before the eruption onset (see e.g. the discussion in Shen et al., 2014a). Quiescent prominences are also very dynamic, being a subject to MHD oscillations (Arregui et al., 2012), such as transverse oscillations, for example triggered by a global coronal wave (e.g. Hershaw et al., 2011; Asai et al., 2012), and longitudinal oscillations (e.g. Vršnak et al., 2007; Zhang et al., 2012; Luna et al., 2014). In turn, based on the direction of the filament main axis displacements, transverse oscillations can have horizontal (e.g. Kleczek and Kuperus, 1969; Hershaw et al., 2011; Shen and Liu, 2012), or vertical polarisations (e.g. Hyder, 1966; Eto et al., 2002; Okamoto et al., 2004; Kim et al., 2014; Mashnich and Bashkirtsev, 2016). Furthermore, quiescent prominence threads are also observed to experience more complicated, chaotic, spatial dynamics during large amplitude oscillations (see e.g. Gilbert et al., 2008; Takahashi, 2017). Complex behaviour of plasma in prominences can be also described in terms of turbulent processes (Berger et al., 2010; Leonardis et al., 2012). Such evidences may be strongly affected by thermodynamic processes acting in prominences, which can also influence the evolution of slow MHD waves (Kumar et al., 2016;

* Corresponding author.
E-mail address: D.Kolotkov@warwick.ac.uk (D.Y. Kolotkov).
[1] Now at Georg-August-Universität Göttingen, Germany.

https://doi.org/10.1016/j.jastp.2018.03.005
Received 9 January 2018; Received in revised form 7 March 2018; Accepted 10 March 2018
Available online 14 March 2018

Ballester et al., 2016). In addition, Kelvin–Helmholtz instability may take place during oscillations of prominences, sustaining damping and plasma heating (Antolin et al., 2014; Terradas et al., 2016). Also, the presence of continuous transverse oscillations in prominences (Hillier et al., 2013) may also be referred to as a self-oscillatory process caused by the interaction of plasma nonuniformities with a quasi-steady flow (Nakariakov et al., 2016).

The equilibrium of prominences is thought to be of a magnetic origin with the Lorentz force counteracting the gravity. In turn, gradient pressure forces can provide an additional support. Considering this basic idea, the following two-dimensional (2D) models of the prominence equilibrium are the most popular: the Kippenhahn–Schlüter (KS, Kippenhahn and Schlüter, 1957) and the Kuperus–Raadu models (KR, Kuperus and Raadu, 1974). The KS model considers the prominence as a plasma slab embedded in the straight magnetic field lines with a dip created by some external sources (e.g. photospheric currents). The magnetic dip outlines a region of magnetic polarity inversion, which justifies a general empirical evidence that prominences lie along the polarity inversion line (also called a neutral line) of large extended bipolar regions (e.g. Bosman et al., 2012). In the KR model the prominence is assumed to be a straight current-carrying horizontal wire located at some height above the conductive photosphere. The support against the gravity is provided by an upward magnetic force acting on the prominence and caused by a virtual "mirror" current, which is located below the photosphere and strictly symmetrical to the prominence. Interestingly, the magnetic topology associated with the KR model resembles that of a coronal cavity, that is a large quasi-circular structure observed off limb in the extreme ultraviolet (EUV) band, and containing a prominence in its interior (Habbal et al., 2010; Gibson et al., 2010).

In the last decades, starting from these two seminal works of KS and KR, a number of studies of 2.5D and full 3D models of prominences have been carried out, taking into account such observational aspects as the presence of a current-aligned magnetic field component, magnetic chirality, "barbs" or "feet" connecting the prominence to the photosphere, Hα fibrils, flows, and their association to CMEs in case of eruptions. In this context, modelling of prominences supported in twisted flux tubes (magnetic flux ropes) by linear force-free field was undertaken by Aulanier and Demoulin (1998) and Aulanier et al. (1998), addressing the natural presence of lateral feet and fibrils. A further approach is to consider extrapolations from photospheric magnetic field data, and compare measurements of prominence locations with the local dips in the resulting coronal magnetic field configurations (Aulanier and Démoulin, 2003; Su and van Ballegooijen, 2012). Blokland and Keppens (2011) studied magneto-hydrostatic (MHS) equilibria for prominences by reducing the MHS equations to an extended Grad–Shafranov equation, and then numerically investigated the spectra of the oscillating structure. A relaxation process is another approach to study the effect of support against the gravity by the magnetic field, where the cold and dense prominence plasma is injected into an initially unperturbed background, and the subsequent evolution is studied numerically. Hillier and van Ballegooijen (2013) studied equilibria for two distinct magnetic field structures of an inverse polarity: a simple O-point configuration, and a more complex one with an X-point. In the former case, the magnetic tension of the field lines compressed at the base of the prominence and stretched at its top is able to sustain prominences, while in the latter case a convergence to a prominence equilibrium is not always guaranteed. Terradas et al. (2013) investigated properties of MHD waves in normal polarity prominences embedded in coronal arcades in terms of the relaxation model too. Stable vertical fast and longitudinal slow MHD oscillations were found. Luna et al. (2012) and Kraśkiewicz et al. (2016) also considered prominences of a normal configuration, residing in a dip formed by curved magnetic field lines. The effects of the magnetic field geometry on longitudinal oscillations in prominences were addressed.

Despite their exceptional importance, the KS and KR models separately are not able to provide an exhaustive picture on the transverse oscillations observed in prominences. For example, the KR model alone allows only for vertically polarised oscillations, while in the pure KS model horizontally polarised oscillations cannot coexist with the vertically polarised ones since the system becomes unstable (van den Oord et al., 1998). A synthesis of these two models, that is a prominence embedded in a magnetic field dip generated by two photospheric currents, accounting also for the effects of the prominence current interaction with the conducting photosphere (via the inclusion of the mirror current effect), has been recently developed in Kolotkov et al. (2016, KNN16). The prominence has been modelled as a line current located above the photosphere at a given height, thus being subject to the gravity and Lorentz forces, which are attributed to the interaction between the photospheric and prominence currents. Such a magnetostatic model, despite its simplicity, provides straightforward results on the prominence dynamics. In KNN16, horizontally and vertically polarised transverse oscillations have been analysed in the linear regime, the equations of motion analytically derived, and dependence of the oscillation properties (e.g. the period) upon the parameters of the system (e.g. the currents in the prominence and at the photosphere) has been determined. In addition, investigation of the mechanical stability of the system shows that the prominence can be stable simultaneously in both horizontal and vertical directions for a certain range of parameters.

In this work, we study oscillations of finite amplitude in terms of the KNN16 model, addressing two main issues: determining the domain of the applicability of the linear approximation derived in KNN16, and responding to the observational detection of finite amplitude oscillations in prominences (e.g. Tripathi et al., 2009). We show that the equations of motion in the vertical and horizontal directions are nonlinearly coupled with each other, in contrast to the linear regime where the motions are essentially independent of each other. Therefore, the presence of nonlinear terms in the governing equations makes the dynamics of the system more various and rich. The paper is structured as follows: in Sect. 2 we present the model and the governing equations; in Sect. 3 we provide an analytical treatment of the equations of motion along the vertical and horizontal directions in the presence of a weak nonlinearity, in Sect. 4 we present an analysis of the oscillation amplitudes and periods by the consideration of a total energy of the system. Finally, discussion and conclusions are provided in Sect. 5.

2. Model and governing equations

Consider a prominence as a horizontal line current i, located at the height h above the plane photosphere in a magnetic dip produced by two spatially separated photospheric line currents of the same strength I parallel to the prominence current, with d being the half-distance between them (see Fig. 1, where the origin of the coordinate system coincides with the centre of the equilibrium current in the unperturbed prominence). The magnetic configuration shown in Fig. 1 corresponds to a normal polarity prominence, i.e. the polarity of the magnetic field lines threading the prominence material coincides with that of the underlying photospheric field (cf. Fig. 2 in Low and Zhang, 2002). Although prominences of this type constitute about 10%–25% of the observed prominences (see e.g. Leroy et al., 1984; Bommier et al., 1994; Parenti, 2014; Ouyang et al., 2017), the flux ropes with a normal configuration are usually observed in the vicinity of active regions (see e.g. Okamoto et al., 2008; Guo et al., 2010; Kuckein et al., 2012; Sasso et al., 2014), and can be responsible for fast CMEs (Low and Zhang, 2002). The horizontal equilibrium of the prominence in such a magnetic system is provided automatically because of the horizontal symmetry of the model, while the vertical equilibrium is determined by the balance of the gravity force $\mathbf{F_g}$ and three Lorentz forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F_m}$ acting on the prominence from the external photospheric and mirror currents, respectively. In the projection onto the z-axis, the vertical equilibrium condition is

$$\frac{2k_1h}{d^2+h^2}+\frac{k_2}{2h}=\mathscr{R}g, \quad (1)$$