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Evaluation of momentum flux with radar

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ABSTRACT

The statistics of gravity wave momentum flux estimation are investigated using data from the MU radar at Shigariki, Japan (136°E, 35°N). The radar has been operating during campaign periods since 1986. The first part of the paper focuses on a multi-day campaign during October 13–31, 1986. The second part of the paper investigates data after 2006 when the radar was operated in a meteor scatter mode. Momentum fluxes are derived from both the turbulent scatter and the meteor scatter measurements, but the techniques are quite different. Probability Distribution Functions are formed using turbulent scatter data. These show that wave packets sometimes have momentum flux magnitudes in excess of 100 $m^2\,s^{-2}$. The technique for meteor radars, introduced by Hocking (2005), has been widely adopted by the radar community in recent years. The momentum flux estimated using this technique is found to be anti-correlated with the background tidal winds. A validation investigation is carried out for periods with a high meteor echo data rate. The conclusion was that the method can be used to calculate the sign of momentum flux, but does not accurately specify the magnitude.

1. Introduction

Internal gravity waves play a crucial role in the mesosphere. They may be generated through convection, or wind shear. The exponential decrease of the atmospheric neutral density with height, and considerations of energy conservation, suggested to early researchers that the effects of upwardly propagating gravity waves were likely to become significant at upper heights (Gossard, 1962; Hines, 1972; Lindzen, 1968; Bretherton, 1969). This expectation has been confirmed by many recent studies (e.g., Yiğit et al., 2009). The effects include a major role in the momentum and energy budget of the upper atmosphere (e.g., Holton, 1982; Yiğit and Medvedev, 2015).

Momentum flux is used to quantify the vertical transfer of horizontal momentum to higher layers in the atmosphere. This parameter is particularly useful since it is conserved for a wave that is propagating *conservatively*. Eliassen and Palm (1961) showed that in the absence of friction and heating, small amplitude (linear), stationary (time-independent) waves in a vertical shear flow are not able to alter the mean flow. This means that the momentum flux is constant with height. This condition, called the non-acceleration theorem is applicable to all vertically propagating atmospheric waves. When conservative conditions, as defined above, are not present, the vertical derivative of momentum flux can be used to quantify a horizontal force per unit mass exerted on the atmosphere where the waves are dissipating.

Vincent and Reid (1983) pioneered a *coplanar-beam* technique whereby ground-based radar can be used to measure momentum fluxes experimentally. In the case of radar studies the momentum flux *density*, i.e., the momentum flux per unit mass (or flux density) is usually reported with units of $m^2\,s^{-2}$ Mesospheric measurements require a high radar system aperture power product which has limited observations to few facilities such as the Buckland Park Radar, Australia (e.g., Vincent and Reid, 1983; Vincent and Fritts, 1987; Fritts and Vincent, 1987), the Saura Radar, Andenes, Norway (e.g., Placke et al., 2014) the MU Radar, Japan (Fritts et al., 1990; Tsuda et al., 1990), the Arecibo Observatory, Puerto Rico (e.g., Janches et al., 2006; Fritts et al., 2006); and the Jicamarca Observatory, Peru (e.g., Riggin et al., 1997). Except for the HF systems (Buckland Park and Saura), the other VHF radars are incapable of making dual-beam wind measurements at night.

Hocking (2005) introduced a new approach to making momentum flux measurements using meteor radar. Such radars can be comparatively modest in size and cost, and can make round the clock observations of the winds. The technique has been widely adopted (e.g., Antonita et al., 2008; Fritts et al., 2010, 2012; Placke et al., 2011a,b; 2015; Andrioli et al., 2013; Liu et al., 2013; de Wit et al., 2015). However, there have been some lingering concerns as to whether statistically meaningful estimates of momentum fluxes can be made over timescales of interest to modellers (Vincent et al., 2010). If a month of averaging is required the results are limited to climatological studies. This question of averaging time

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will be explored later in this paper, along with a more general discussion of momentum flux estimation.

2. Theoretical considerations

2.1. Coplanar beam technique

We begin by looking at the elementary properties of momentum flux for the symmetric beam configuration as first proposed by Vincent and Reid (1983). For the MU Radar experiments the radar was transmitting in narrow coplanar beams, which were at a zenith angle of $\chi = 10°$ from the vertical. The radial winds were measured in the four cardinal directions with an additional beam in the vertical. Turbulent backscatter from a VHF radar like MU (which operates at 45.6 MHz) is only obtained during the daytime. The data typically have numerous random gaps, but they are fewer in number near local noon when the photo-ionization is at a maximum, and the height with the maximum number of detections is ~73 km. Our analysis focuses on this height where the statistics are optimal, although higher altitudes might be of more interest from a geophysical standpoint where waves are breaking and dissipating. Considering one pair of symmetric coplanar beams, the radial velocity at the positions of the two echoing regions is symbolized by $\acute{r}_1$ and $\acute{r}_2$, respectively. The $\acute{r}$ symbols denote perturbation velocities, i.e., the time mean and low frequencies have been removed, including the diurnal and semidiurnal tides. A high-pass filter with a band edge of two hours was used for the MU analysis. Assume that (u_1, w_1) and (u_2, w_2) are the orthogonal horizontal and vertical wind component velocities at coplanar beam positions 1 and 2, respectively. With χ being the zenith angle we can write,

$$\acute{r}_1 = \hat{u}_1 \sin\chi + \hat{w}_1 \cos\chi \tag{1}$$

$$\acute{r}_2 = -u_2 \sin\chi + w_2 \cos\chi. \tag{2}$$

From these equations we can solve for the estimate $\hat{u}$ and $\hat{w}$

$$\hat{u} = \frac{\acute{r}_1 - \acute{r}_2}{2\sin\chi} \tag{3}$$

$$\hat{u} = \left(\frac{u_1 + u_2}{2}\right) + \cot\chi\left(\frac{w_1 - w_2}{2}\right) \tag{4}$$

$$\hat{w} = \frac{\acute{r}_1 + \acute{r}_2}{2\cos\chi} \tag{5}$$

$$\hat{w} = \tan\chi\left(\frac{u_1 - u_2}{2}\right) + \frac{w_1 + w_2}{2} \tag{6}$$

The estimators, $\hat{u}$ and $\hat{w}$, are biased estimators, as are the respective variances. This can be seen since the equation for $\hat{u}$ contains $\hat{w}$ terms and the equation for $\hat{w}$ contains u terms. These extra terms apply a spatial filtering to the data. If the gravity wave spectrum contains waves with a horizontal wavelength sufficiently short, the phase difference between the coplanar beams will produce a bias. In fact, we found that the vertical velocity fluctuations derived from the east/west beam pair or the north/south beam pair were systematically larger than the velocity amplitudes determined from the vertical beam. Horizontal wave motions are therefore projecting into the vertical wind estimates made with the oblique beams and causing a positive bias.

The momentum flux only has physical meaning when averaged. In radar data the average is in time and sometimes indicated by the angular brackets. However, from the equations above we can write the instantaneous value as

$$\hat{u}\hat{w} = \frac{\acute{r}_1^2 - \acute{r}_2^2}{4\sin\chi\cos\chi} \tag{7}$$

$$\hat{u}\hat{w} = \frac{u_1 w_1}{2} + \frac{u_2 w_2}{2} w_0 + \tan\chi\left(\frac{u_1^2 - u_2^2}{4}\right) + \cot\chi\left(\frac{w_1^2 - w_2^2}{4}\right). \tag{8}$$

The estimator for momentum flux is a noisy estimator in that there are spurious terms, i.e., the term involving $u_1^2 - u_2^2$ in Eq. (8). It is an unbiased estimator insofar as the result is insensitive to a phase difference between beams 1 and 2. However, the spurious terms may have more control over the length of time required to obtain a reliable estimate than the low degree of correlation between $\hat{u}$ and $\hat{w}$, or observational errors in the measurement.

Vincent and Reid (1983) describe the momentum flux as "... the difference in the mean squared *radial* velocities….". Mathematically the momentum flux according to this view is

$$\frac{1}{N}\sum_{i=0}^{N-1}\hat{u}_i\hat{w}_i = \frac{1}{4\sin\chi\cos\chi}\left(\frac{1}{N}\sum_{i=0}^{N-1}\acute{r}_1^2 - \frac{1}{N}\sum_{i=0}^{N-1}\acute{r}_2^2\right). \tag{9}$$

However, if we directly apply Eq. (7), we obtain a slightly different estimator.

$$\frac{1}{N}\sum_{i=0}^{N}\hat{u}_i\hat{w}_i = \frac{1}{4\sin\chi\cos\chi}\frac{1}{N}\sum_{i=0}^{N}\left(\acute{r}_1^2 - \acute{r}_2^2\right). \tag{10}$$

It can be easily shown that the estimators of (9) and (10) are different. Assuming we have time series of $\acute{r}_1^2$ and $\acute{r}_2^2$, the order of each of these can be randomized. The numerical result of Eq. (9) will be identical regardless of the order. The numerical result of (10) will not be identical for different ordering. This difference does not necessarily imply that the momentum flux estimate given by (9) is biased. However, the extent to which the (10) estimate depends on the ordering of the data values may imply the susceptibility of the estimate based on Eq. (9) to bias. Eq. (10) is a literal representation of covariance, and in that sense is more mathematically rigorous. It is possible that Eq. (9) could give a statistically more stable result when the data acceptance rate is low. This is because radial velocity estimates could be included in the variance, even when a valid detection was not made on the other beam of the coplanar pair.

Eqs. (9) and (10) would be equally valid if the quantity being averaged is a stationary *Gaussian* random variable. They may also give highly similar results with the real processes encountered in the atmosphere. This will be investigated in a later section. For some of the radar experiments at Jicamarca (Riggin et al., 1997) the momentum flux was computed using the (10) estimator. For the experiments at Buckland Park (e.g., Vincent and Reid, 1983) Eq. (9) was used. For some MU radar experiments Eq. (9) was applied to one-hour time segments (Toshitaka Tsuda, private communication) and these were ensemble averaged to create daily estimates. This was a reasonable compromise, since stationarity is more likely to be maintained over shorter time averages.

2.2. Hocking method theory

The Hocking (2005) technique for estimating momentum flux with meteor radar is an appealingly simple concept. However, a large matrix needs to be inverted to obtain the momentum flux components. The technique will not be fully described here, since