Journal of Atmospheric and Solar-Terrestrial Physics 105-106 (2013) 281–286

Contents lists available at ScienceDirect

Journal of Atmospheric and Solar-Terrestrial Physics

journal homepage: www.elsevier.com/locate/jastp

Radar baud length optimisation of spatially incoherent time-independent targets

Markku S. Lehtinen [a,*], Baylie Damtie [b]

[a] Sodankylä Geophysical Observatory, Tähteläntie 62, FIN-99600 Sodankylä, University of Oulu, Finland
[b] Washera Geospace and Radar Science Laboratory, P.O. Box 79, Bahir Dar University, Bahir Dar, Gojjam, Ethiopia

ARTICLE INFO

Article history:
Received 17 February 2012
Received in revised form
3 October 2012
Accepted 9 October 2012
Available online 7 November 2012

Keywords:
Radar
Perfect codes
Optimal baud length
Extended target

ABSTRACT

While it may be a general belief that the optimal baud length for radar measurements of range extended targets should be close to the desired resolution, this is only an approximate truth for weak targets and not true at all for strong targets. We use full measurement error estimates with proper correlations and find numerically the baud length which optimises the posteriori variance of an extended target. While the pulse is assumed to be a simple boxcar with a given fixed energy and the baud length is the only design parameter, the results extend to many traditional ways of pulse compression coding through arguments derived from recent results on rigorous experiment comparison and perfect coding.

1. Introduction

The problem of designing an optimal radar waveform for various applications has attracted the attention of many researchers since the publication of a widely cited work on theory of ambiguity function by Woodward (1953). A book that totally focusses on radar waveform design and analysis has been published a long time ago (Cook and Bernfeld, 1967). Also Levanon and Mozeson (2004) have published a more recent textbook on the subject. The output SNR from a radar receiver is often used as a metric in the evaluation of the performance of a radar waveform. In this measure, an optimal radar waveform is a waveform that gives the maximum possible output SNR. Such a waveform can found in practice by carrying out an exhaustive search as shown, for example, Rohling and Plagge (1989) and Ruprecht and Rupf (1996). A similar investigation has been carried out by the present authors (Lehtinen et al., 2004) in order to find an optimal radar waveform and receiver impulse response pairs that give maximum possible SNR without creating unwanted sidelobes.

Even though the output SNR is a very popular measure of the performance a radar waveform, it is hard to find a mathematical theorem to show that a maximum SNR directly translates into a maximum gain of information, where information is defined in a wider sense than mere detection. By recognising this problem and to design an optimal radar waveform suitable for extracting a maximum information as well as detection, Bell (1993) has introduced the use of information theory. This work treats the problem of optimising a radar waveform for optimal inference of information and detection (SNR optimisation) as two separate optimisation problems. They are solved separately. For example, a radar waveform designed for obtaining maximum information is obtained based on an extended radar target with a random impulse response and then optimising the mutual information between the target and the radar waveform. In general, a radar waveform optimisation problem is application-specific and often the desired information about a radar target and also the characteristics of the associated unwanted signals that corrupt the measurements determine the nature of the corresponding optimal radar waveform.

The temporal and spatial resolutions of a measurement are usually the major concern in the design of an optimal radar waveform for incoherent scatter radars (for details see Lehtinen et al., 2009 and the references within). Application of statistical inversion for lag profile estimation in radars in general was described in Lehtinen (2001). Damtie et al. (2004) showed how one can use the Bayesian statistical inversion technique in the deconvolution of the lag profile of an incoherent scatter radar measurement. This method is very flexible and capable of producing a very high-resolution measurement. The analysis is carried out by considering the measurements, the unknown ionospheric plasma lag profiles and the unwanted noise as random variables. The method has been further developed for practical use in extracting ionospheric parameters by Virtanen et al. (2008, 2009)

* Corresponding author. Tel.: +358 400399982; fax: +358 16619875.
E-mail addresses: markku.lehtinen@mac.com, markku.lehtinen@sgo.fi (M.S. Lehtinen).

1364-6826/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.jastp.2012.10.010

and Nikoukar et al. (2008). It is useful to notice that the measured lag profiles are the convolutions of the target scattering autocorrelation functions with the range ambiguity function (instrument function). This means that the target estimation can be understood as a deconvolution problem. Also, if the target autocorrelation function is only a weak function of the lag measured, many different convolution kernels are available as independent convolution measurements and it follows that the useful target resolutions become independent of the baud lengths used in the radar codes.

In the case of inversion analysis we need a different metric to measure the performance of a radar waveform and to be able to choose the optimal ones for this kind of work. In the design of an optimal radar waveform of an incoherent scatter radar experiment employing lag profile deconvolution by means of Bayesian inversion, the natural metric to use is the a posteriori variance of the inverted lag profiles. This means that a radar waveform which gives the minimum possible variance can be considered as an optimal waveform. Phase-coded radar waveforms have been used in the inversion analysis, and the corresponding a posteriori variance depends on the phase patterns and the baud lengths of these waveforms. Lehtinen et al. (2008) have presented a fast method of theoretically comparing the posteriori variances produced by these radar waveforms with different phase patterns which may be used in choosing the optimal sequences for practical use in an incoherent scatter radar. The baud length optimisation has been investigated by Lehtinen (1989) for the case of incoherent-scatter measurements with only a very low SNR.

In the present paper we focus on the choice of an optimal baud length of a radar waveform in an incoherent scatter radar experiment with a given desired range resolution and any level of SNR, assuming the total pulse energy is constant. The analysis is based on a series of numerical computations of the posteriori variances for the corresponding inverted lag profile by considering many different combinations of baud length and SNR. The range resolution is assumed to be one. As the covariance calculation numerics is very expensive, different algorithms and approximations are necessary for different combinations of baud length and SNR, but a unified picture of the dependence can be found by collecting the results of these calculations together.

The target power is described by a normalised SNR denoted by nSNR, corresponding to the SNR we would get by using a pulse length equal to the (fixed) target resolution (assumed to be one) and a receiver filter also corresponding to this. The actual SNR is of course a function of our optimisation parameter – the pulse length – and thus not a good measure of the target power.

2. Review of an incoherent-scatter signal model

We model the incoherent scatter radar signal (considering the ionosphere as a time-independent and continuously distributed target for our purpose) by the Itō integral scattering relation (Van Trees, 1971; Lehtinen et al., 2009)

$$z^q(t) = \int \epsilon^q(t-r)\mu^q(dr) + \sqrt{T}\xi^q(t), \tag{1}$$

where ϵ^q is the transmission envelope, ξ^q is the white-noise process with T denoting the equivalent noise temperature, and μ^q is the complex Itō measure describing the scattering of a CW carrier from elementary range cells dr of the target. The index q denotes independent repetitions of a radar experiment and r is the range in terms of time. We consider a radially stratified target (or alternatively consider angular dependencies integrated away) and thus the Itō measure varies only along the radial direction. The derivations are also valid for a bistatic case, if we denote by r the distance from the transmitter through the scattering point to the receiver. In this work, the random variables μ and ξ are assumed to be complex Gaussian, zero-mean and also mutually independent. Complex Gaussian also implies that expectations of products (second moments), where neither of the product terms is conjugated, are zero. In addition, we assume that all random variables with $q \neq q'$ are independent.

The spatial correlation of this Itō measure signal may be described by

$$\langle \mu^q(dr)\overline{\mu^q(dr')} \rangle = \sigma(r)\delta(r-r')\, dr\, dr', \tag{2}$$

where $\sigma(r)$ corresponds to the target density (mathematically: the structure function for the Itō measure signal) and $\delta(r-r')$ is the Dirac delta function. This is the mathematically rigorous way to model a spatially incoherent target, where the scatterings from disjoint volumes are mutually independent. If the target were be time-dependent we would have

$$\langle \mu^q(dr;t)\overline{\mu^q(dr';t')} \rangle = \sigma(r;t-t')\delta(r-r')\, dr\, dr', \tag{3}$$

with $\sigma(r,\tau)$ being the target autocorrelation function depending on time lag $\tau = t-t'$, but for the present case, the simpler formulation is sufficient. In addition, the temporal correlation of the white-noise process is given by

$$\langle \xi^q(t)\overline{\xi^q(t')} \rangle = \delta(t-t'). \tag{4}$$

If the transmission envelope is of a unit length and unit power and the scattering power associated with the plasma autocorrelation function is also of a unit length, the temperature T of the thermal noise can be identified with the inverse of SNR, that is $T = \mathrm{SNR}^{-1}$. In many applications and especially here, it is not necessary to assume that the code changes between the independent repetitions of the experiment. If this is the case we can denote $\epsilon = \epsilon^q$ for all q. The lag estimate of our measurements can then be given by

$$\langle z^q(t)\overline{z^q(t')} \rangle = \int\int \epsilon(t-r)\overline{\epsilon(t'-r')}\langle \mu^q(dr)\overline{\mu^q(dr')} \rangle + T\langle \xi(t)\overline{\xi(t')} \rangle. \tag{5}$$

Using the expressions in Eqs. (2) and (4) in the place of the corresponding relations in Eq. (5) and carrying out the integration along r', we get

$$\langle z^q(t)\overline{z^q(t')} \rangle = \int \epsilon(t-r)\overline{\epsilon(t'-r)}\sigma(r)\, dr + T\delta(t-t'). \tag{6}$$

3. Discrete representation of the plasma autocorrelation function and modulation envelope

The corresponding discrete formulation of the measurement model is necessary in order to investigate the accuracies of different baud lengths and desired range resolution combinations by considering different kinds of SNR scenarios. This means that we need to discretise the underlying plasma autocorrelation function (scattering model) and the modulation envelope. This can be done in practice by using an appropriate basis function. For the purpose of this paper, we can use the staircase spline basis function defined by

$$\psi(r) = \begin{cases} 1 & \text{if } 0 \le r < 1, \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

It is also possible to use the triangular spline basis

$$\psi(r) = \begin{cases} r & \text{if } 0 < r < 1, \\ 2-r & \text{if } 1 \le r < 2, \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

One may also use smoother spline basis functions.