Journal of Atmospheric and Solar-Terrestrial Physics 102 (2013) 8–16

Contents lists available at ScienceDirect

Journal of Atmospheric and Solar-Terrestrial Physics

journal homepage: www.elsevier.com/locate/jastp

Non-linear whistler mode wave effects on magnetospheric energetic electrons

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ARTICLE INFO

Article history:
Received 9 July 2012
Received in revised form
12 March 2013
Accepted 16 March 2013
Available online 23 April 2013

Keywords:
Nonlinear plasma wave
Energy transport
Pitch angle change

ABSTRACT

Electron energy transport due to nonlinear plasma-wave and particle interactions is carried out by waves and particles resonating with each other. Many previous nonlinear wave studies have only considered the main resonance between waves and electrons, since the contributions from other resonant orders were ignored as insignificant. We have found through test particle simulations, however, that although independent separate contributions from higher-order resonances can be small, they can have a rather significant impact on the main-order contribution and hence on the total nonlinear wave effects. Contributions from different orders can interfere with each other and the overall nonlinear wave effect is significantly different from that of just the major resonance. Therefore, in the nonlinear wave/particle interaction regime, contributions from different resonant orders are inseparable and contributions from higher order wave–particle resonances should all be included. Similarly, banded plasma waves should be used in nonlinear wave studies instead of assuming monochromatic waves. When the essential factors mentioned above are included, the overall electron transport due to the nonlinear plasma wave effects takes the form of a diffusion-like process, rather than advection, as reported in many previous studies. It is also found that electron transport induced by whistler mode waves is an important mechanism for the formation of the electron butterfly pitch-angle distribution.

1. Introduction

The effects of plasma waves on the Earth's radiation belt electrons are often approximated by electron pitch angle and energy diffusion, whose associated diffusion coefficients can be calculated using the quasi-linear theory (Lyons, 1974; Glauert and Horne, 2005; Summers, 2005). A few assumptions have to be employed in order to apply the quasilinear theory to wave–particle interactions. The first is that the wave amplitude is relatively small; second, contributions from different wave–particle resonant orders are independent of each other (the main point of this study) and third, wave damping or growth rates cannot be large compared to the real part of the wave mode frequency ω_r. When the wave amplitude is large enough, non-linear effects such as electron phase bunching and phase trapping (Roth et al., 1999, Albert, 2000, 2002; Omura and Summers, 2006; Albert and Bortnik, 2009; Bortnik et al., 2008; Bortnik and Thorne, 2010; Liu et al., 2010; Tao et al., 2011) can occur when plasma waves resonate with electrons under certain circumstances—for example, when the waves and the electrons are in phase for a long time and the wave amplitude is relatively large; these non-linear effects are usually more efficient in accelerating the electrons to much higher energy level or scattering them into the loss cone (Albert and Bortnik, 2009; Bortnik et al., 2008; Bortnik and Thorne 2010). Nonlinear wave effects therefore need to be addressed for the sake of both scientific purposes and global space weather modeling and forecasting.

The test particle method is a powerful tool for the investigation of the nonlinear plasma wave–particle interactions. Numerous studies had been conducted using test particle methods since the 1960s (Bell, 1965, 1984; Dysthe, 1971; Nunn, 1971; Omura and Matsumoto, 1982; Omura and Summers, 2006; Omura et al., 2007; Inan et al., 1978; Bortnik and Thorne, 2010; Albert and Bortnik, 2009; Liu et al., 2010). Electron interactions with major plasma waves in the magnetosphere such as chorus, electromagnetic ion cyclotron (EMIC), and magnetosonic waves were investigated using the test particle method. Nonlinear effects are clearly visible when wave amplitudes are larger than a certain threshold value. When wave amplitudes are small and quasilinear theory can be applied, energy transport is dominated by electron energy diffusion; and the corresponding diffusion coefficients obtained by

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http://dx.doi.org/10.1016/j.jastp.2013.03.018

quasilinear theory and test particle method agree perfectly (Tao et al., 2011).

In the aforementioned studies, however, test particle simulations of electron–wave interactions primarily focused on electron–wave interactions during just one small fraction of a magnetic field line along which the test electrons bounce back and forth. An automatic question one would raise: what is the long-term accumulative effects on electron energy transport imposed by plasma waves? In our recent study (Zheng et al., 2012), we extended the wave–electron interaction time and considered multiple passes (many bounce periods) of electrons through the wave region to see the long-term cumulative effects of wave–particle interactions on electron behavior. Whistler-mode waves were selected because of their common occurrences in the near-Earth region, its potential significance to radiation belt dynamics and diffuse auroral precipitation (Ni et al., 2011a,b), their narrow-banded wave spectrum and the nonlinear wave–particle interactions often associated with them (Yoon, 2011; Wilson et al., 2011; Summers et al., 2011). With an assumed monochromatic, field-aligned whistler mode wave and only first order counter-streaming (i.e., $n=-1$) resonance being considered, we found that electron energy transport was more like energy advection instead of diffusion and the electron energy of test electrons tended to grow larger, while the electron pitch angles tended to decrease, inversely correlating the energy advection—which coincides with the simulation results reported by Roth et al. (1999).

We extend our previous study (Zheng et al., 2012) by conducting test particle simulations for more general, realistic cases to investigate nonlinear plasma wave effects by including higher orders or wave–particle resonances and employing banded whistler mode waves instead of monochromatic waves. In our previous study, we found that the electron energy transport by waves is sensitive to η, the phase angle between the perpendicular component of the electron velocity vector and the wave magnetic field, because small perturbation of the electron phase would affect the phase trapping time, which is highly related to energy advection. The energy of test electrons will not change much if the electrons are not in resonance with the plasma waves. The majority of previous test particle investigations – including one of ours – were based on nonlinear wave effects based on a set of seemingly reasonable assumptions such as only the main resonance being included, the wave being monochromatic and field aligned, etc. In fact, some of the assumptions may not hold in more realistic cases because of the sensitivity of energy transport to electron phase change. Therefore, it is possible that these assumptions could make much of the simulation results appear more heuristic than being physically sound, as we will demonstrate later in our simulation results.

2. Test particle method

The equation of motion of a charged relativistic particle (electron in this work) in a magnetic field is

$$\frac{dp}{dt} = q_e\left\{E^w + \frac{p}{m_e\gamma}\times\left[B^w + B_0(\lambda)\right]\right\} \tag{1}$$

where $p=\gamma m_e v$ is the electron momentum with $\gamma = c/\sqrt{c^2-v^2}$ the Lorentz factor; m_e the particle rest mass; $\mathbf{v}$ the velocity vector; and B, and E the wave magnetic and electric fields, respectively. B_0 is the Earth's magnetic field, which is approximated by a dipole field in this study. The amplitude of B_0 can be expressed as

$$B_0(L,\lambda) = B_{\mathrm{Earth}}\sqrt{(1+3sin^2\lambda)}/(L^3cos^6\lambda) \tag{2}$$

Since we are interested in the electron–wave interaction in Earth's dipole magnetic field, in which electrons undergo gyromotion along the Earth's magnetic field line, it is advantageous to solve Eq. (1) in a cylindrical coordination system along the field line (Bell, 1984; Tao and Bortnik, 2010).

Assuming the whistler mode wave field takes the form

$$B_w = \tilde{B}_w e^{i\Psi}, \tag{3}$$

where $\Psi \equiv \int \mathbf{k}\cdot d\mathbf{r} - \int \omega dt$ is the wave phase, the wave electromagnetic field amplitudes can be written as

$$E^w = -\hat{\mathbf{e}}_x E^w_x \sin\Psi - \hat{\mathbf{e}}_y E^w_y \cos\Psi - \hat{\mathbf{e}}_z E^w_z \sin\Psi$$
$$B^w = \hat{\mathbf{e}}_x B^w_x \cos\Psi - \hat{\mathbf{e}}_y B^w_y \sin\Psi + \hat{\mathbf{e}}_z B^w_z \cos\Psi \tag{4}$$

The wave electromagnetic field components are not independent and can be related by the plasma dispersion relation (Stix, 1992).

$$E^w_x/B^w_y = \frac{c(P-N^2sin^2\theta)}{NPcos\theta}$$
$$E^w_y/B^w_y = -\frac{cD(P-N^2sin^2\theta)}{NPcos\theta(S-N^2)}$$
$$E^w_z/B^w_y = -\frac{cNsin\theta}{P}$$
$$B^w_z/B^w_y = \frac{Dsin\theta(P-N^2sin^2\theta)}{Pcos\theta(S-N^2)} \tag{5}$$

where D, S, and P are the Stix parameters, $N=|\mathbf{k}|c/\omega$ is the ratio of the velocity of light to the wave phase velocity. $\mathbf{k}=k(\sin\theta, 0, \cos\theta)$ is the wave vector, and θ is the wave normal angle. After some mathematics, we obtain a set of three equations

$$\frac{dp_{//}}{dt} = (eE^w_z J_{-n} - \omega_R p_\perp J_{-(n+1)} + \omega_L p_\perp J_{-(n-1)})\sin\eta - \frac{p_\perp^2}{2m_e\gamma\Omega}\frac{\partial\Omega}{\partial z} \tag{6}$$

$$\frac{dp_\perp}{dt} = [(p_{//}-p_R)\omega_R J_{-(n+1)} - (p_{//}+p_L)\omega_L J_{-(n-1)}]\sin\eta + \frac{p_\perp p_{//}}{2m_e\gamma\Omega}\frac{\partial\Omega}{\partial z} \tag{7}$$

$$\frac{d\eta}{dt} = \frac{n\Omega}{\gamma} + k_{//}v_{//} - \omega \tag{8}$$

where η is the gyroperiod-averaged phase angle between the particle velocity vector in the x–y plane and the right-hand circularly polarized component of the wave magnetic field (Fig. 1), ω is the whistler mode wave frequency, and Ω is the electron gyrofrequency. n is the resonant order of electron gyroresonance, and $J_n = J_n(k_\perp p_\perp/\gamma m_e\Omega)$ is the Bessel function of the first kind with order n. P_L, P_R, ω_L, and ω_R are defined as follows:

$$p_L = \gamma m_e \frac{E^w_x - E^w_y}{B^w_x - B^w_y} \tag{9}$$

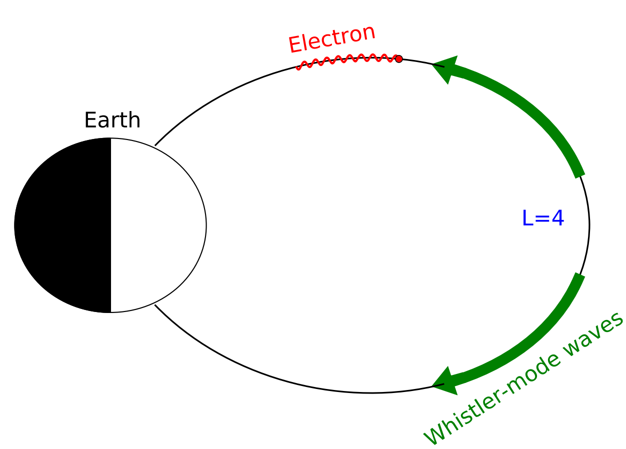

Fig. 1. Diagram of an electron interacting with a chorus wave. The electron bounces back and forth along the magnetic field line between two mirror points.