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Evolution of heliospheric magnetized configurations via topological invariants

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ABSTRACT

The analogy between magnetohydrodynamics (MHD) and knot theory is utilized in presenting a new method for an analysis of stability and evolution of complex magnetic heliospheric flux tubes. Planar projection of a three-dimensional magnetic configuration depicts the structure as a two-dimensional diagram with crossings, to which one may assign mathematical operations leading to robust topological invariants. These invariants enrich the topological information of magnetic configurations beyond helicity. It is conjectured that the field which emerges from the solar photosphere is structured as one of the simplest knots—unknot or prime knot—and these flux ropes are then stretched while carried by the solar wind into the interplanetary medium. Preservation of invariants for small diffusivity and large cross section of the emerging magnetic flux makes them impervious to large scale reconnection, allowing us to predict the observed structures at 1 AU as elongated prime knots. Similar structures may be observed in magnetic clouds which got disconnected from their footpoints and in ion drop-out configurations from a compact flare source in solar impulsive solar events. Observation of small scale magnetic features consistent with prime knots may indicate spatial intermittency and non-Gaussian statistics in the turbulent cascade process. For flux tubes with higher resistivity, magnetic energy decay rate should decrease with increased knot complexity as the invariants are then harder to be violated. These observations could be confirmed if adjacent satellites happen to measure distinctly oriented magnetic fields with directionally varying suprathermal particle fluxes.

1. Introduction

Magnetic fields are encountered at every corner of the universe and their existence imposes far reaching consequences on the evolution of various astrophysical objects. The intergalactic seed magnetic field of $B < 10^{-11}$ G (Kronberg, 2001) is believed to be formed via the battery effect due to non-parallel temperature and density gradients, generating curl of electric field from Ohm's Law (Biermann, 1950; Subramanian et al., 1994), while dynamo action due to plasma turbulence may enhance it by many orders of magnitude. Terrestrial and planetary magnetic fields, formed also via dynamo process, constitute protective shields from hazardous solar and galactic cosmic rays. Stellar magnetic fields are important in the transport and shedding of angular momentum, condition required for slowing down of the spinning star (Bouvier, 2009). Stressed, solar (stellar) marginally stable magnetic field loops (a) form the sites of flares, resulting in the observed X-ray images, and (b) initiate the CME, coronal mass ejections of large plasma bulbs which energize particles over vast distances. Magnetic field plays also a crucial role in driving the solar wind (SW), in the evolution of interstellar turbulence and accretion discs, in formation of jets in the proto-planetary system and young stars, as well as in active galactic nuclei where the magnetic field permeates a rotating black hole.

The origin, formation, structure and decay of magnetic fields in all these astrophysical systems constitute topics of great importance. Particularly interesting are processes which transfer energy, momentum and other physical quantities through evolution of various three-dimensional magnetized configurations. Observations of spatially complex magnetic structures in space, in laboratories and in simulations validate an implementation of fresh methods regarding their description and evolution. The present paper presents a new approach in the classification of magnetic fields through their spatial structure in the realm of magnetohydrodynamics (MHD)–knot theory and beyond, and applies it to observation of SW texture, magnetic clouds, localized magnetic relaxation patches and to decay of magnetic ropes via their topological invariants.

2. Characterization of magnetized plasma

Some of the most important features related to the evolution of magnetized plasma are the various invariants of single particle motion and of the plasma fluid immersed in the magnetic field.

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The single particle invariants are related to the quasi-periodic motions in the presence of a static, inhomogeneous magnetic field, while the macroscopic invariants are determined by the global structure of magnetic field as it interacts with the plasma. The geometrical structure of the magnetic field can be characterized through its topological invariants (Section 3), while interaction between the charged plasma particles constitutes a source of resistivity and viscosity, which affect the decay and the (inverse) cascade of the magnetic fields. Therefore, as the magnetic field, immersed in plasma, modifies its magnitude and its geometrical configuration, the conservation of invariants imposes severe constrains on its evolution. Magnetic helicity is the best known invariant in magnetized plasma (e.g., Berger and Field, 1984). It has topological interpretation as a linkage between two non-twisted isolated flux tubes, while its generalization includes both twist and writhe (Section 3). In a closed ideal MHD system it is a conserved quantity varying only on the slow diffusive timescale. Therefore, it is believed that release or transfer of magnetic energy approximately conserves the magnetic helicity. For example, helicity flux due to photospheric motion feeds the coronal helicity, which is intermittently ejected via CME. Experimental helicity assessment in magnetic cloud (MC), a subset of CME with a twisted magnetic flux tube, which propagates in the interplanetary medium after being released from solar active region (AR), can help in understanding the coronal ejection process and in constraining its theoretical models (e.g., Dasso et al., 2005, 2006). The injection of helicity from a spinning AR into the corona and transfer of helicity to MC is still an unsettled problem (e.g., Ravindra et al., 2011). For wrapped magnetic field configurations, additional invariants have been suggested (Berger, 1990; Monastyrskii and Sasorov, 1987; Ruzmaikin and Akhmetiev, 1994). These invariants, which characterize the structure of the magnetic flux tubes, constrain the relaxation processes for marginally stable plasma, affect the dynamo process and the formation of large scale field, and may explain decay rates or stability of magnetic configurations. Here we present another method for invariants' construction.

Generally, plasma can be divided into an electron fluid which follows changes due to electric field, Lorentz and Hall forces, electron pressure gradient and plasma resistivity η:

$$\frac{m_e}{e}\frac{d\mathbf{v}_e}{dt}+\frac{\mathbf{J}\times\mathbf{B}}{nec}-\frac{\nabla p_e}{ne}+\eta\mathbf{J}=\mathbf{E}+\frac{\mathbf{V}\times\mathbf{B}}{c} \tag{1}$$

and center of the mass fluid which is advanced under the effects of the (total) kinetic and magnetic pressures and magnetic tension (with an additional effect of viscosity)

$$\rho\frac{d\mathbf{V}}{dt}=-\nabla p+\frac{\mathbf{J}\times\mathbf{B}}{c}=-\nabla\left(p+\frac{B^2}{8\pi}\right)+\frac{(\mathbf{B}\cdot\nabla)\mathbf{B}}{4\pi} \tag{2}$$

while the electromagnetic fields are related via Maxwell equation

$$\nabla\times\mathbf{E}=c^{-1}\partial\mathbf{B}/\partial t \tag{3}$$

In the lowest order approximation for the electron fluid one may ignore most of the terms on the LHS of Eq. (1) and insert the result into the Maxwell equation. The electrons now are tied almost entirely to the ions, and without resistivity the magnetic field is frozen into the plasma:

$$\frac{\partial\mathbf{B}}{\partial t}=\nabla\times(\mathbf{V}\times\mathbf{B})+\eta\nabla^2\mathbf{B} \tag{4}$$

This constitutes the approximate evolutionary equation for the (turbulent) magnetic field in the MHD approximation, where both electron and ion fluids move almost in tandem, dragging the magnetic field. The diffusivity is included in the last term, making generally the Reynolds number LV/η extremely large, where L denotes the scale of the magnetic field.

3. Knot–MHD classification.

Magnetic fields can be described in terms of a fictitious set of loops in R^3 which evolve through an interaction with the plasma fluid. In the ideal MHD approximation, in the absence of magnetic diffusivity, the plasma flow drags the magnetic field lines, enabling them only to stretch and bend. Hence, MHD fluid can be visualized as a collection of entangled, non-intersecting, slowly evolving fields. Similarly, mathematical knot is depicted as a closed loop of a non-self-intersecting curve, continuously deformed in R^3, following the laws of knot topology—smooth changes in the surrounding viscous fluid—allowing only stretching or bending. Therefore, MHD evolution may be viewed as a topological deformation of equivalent configurations with an embedded set of invariants; similarly, knots evolve preserving their invariants, while non-equivalent knots are distinguished by different topological invariants. Such invariants are crucial in obtaining information about the knot or a link (collection of non-intersecting, entangled knots), and equivalently, about the (turbulent) magnetic field. The topological information about knots may be obtained from their diagrams—2D planar projections which preserve the over/under-crossing of the 3D curve. A knot without any crossings, termed unknot, is equivalent to a planar loop (circle). In both descriptions two loops are said to be (topologically) *equivalent* if one can be transformed into the other via deformation of R^3 upon itself; these transformations correspond to smooth modifications of one knotted string into another such that in all intermediate stages the loops evolve without self-intersection.

General deformations which satisfy this equivalency (ambient isotopy) were described in the three link moves R_j, $j=1$, 3 (Reidemeister, 1926); they allow manipulation of projections without changing the knot structure. These basic moves include: twist (green), poke (orange) and slide (Fig. 1). More trivial deformation like simple loop wiggling is denoted by R_o. Knot invariant is obtained by assigning to each of its crossings specific mathematical operation and self-consistency over the whole knot results in a unique numerical value or algebraic function which will not change when the knot undergoes the R_j deformations. Fig. 2 shows two configurations with assigned value of a *writhe* ω, which takes the values of ± 1, depending on knot orientation. The writhe is an invariant only under R_2 and R_3. Similar procedure regarding a link of two knots α, β, with summation over all the

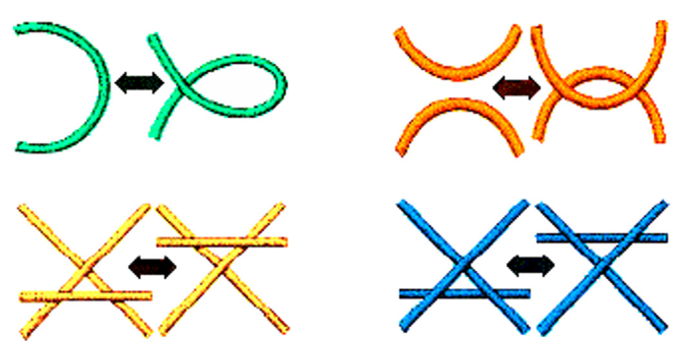

Fig. 1. Reidemeister moves. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

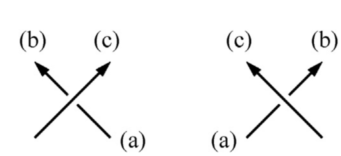

Fig. 2. Writhe: [+] (left) and [−] (right).