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Theory of multiple-stellar population synthesis in a non-Hamiltonian setting

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ABSTRACT

We aim to investigate the connections existing between the density profiles of the stellar populations used to define a gravitationally bound stellar system and their star formation history: we do this by developing a general framework accounting for both classical stellar population theory and classical stellar dynamics. We extend the work of Pasetto et al. (2012) on a single composite-stellar population (CSP) to multiple CSPs, including also a phase-space description of the CSP concept. In this framework, we use the concept of distribution function to define the CSP in terms of mass, metallicity, and phase-space in a suitable space of existence $\mathbb{E}$ of the CSP.

We introduce the concept of foliation of $\mathbb{E}$ to describe formally any CSP as sum of disjointed Simple Stellar Populations (SSP), with the aim to offer a more general formal setting to cast the equations of stellar populations theory and stellar dynamics theory. In doing so, we allow the CSP to be object of dissipation processes thus developing its dynamics in a general non-Hamiltonian framework.

Furthermore, we investigate the necessary and sufficient condition to realize a multiple CSP consistent with its mass-metallicity and phase-space distribution function over its temporal evolution, for a collisionless CSP. Finally, analytical and numerical examples show the potential of the result obtained.

1. Introduction

Stars are the fundamental constituents of a galaxy. Our understanding of galactic structure and evolution depends very much on the processes governing their birth, and evolution. The evolutionary time scales of stars, their energy feedback, yields of chemically enriched material into the interstellar medium, end products of their evolutionary history, and distribution in space and time characterize the structure of the galaxies and govern their evolution. However, all these stellar phases and products are often subject to uncertainties of both theoretical and observational nature, generating a lacking comprehension of these important issues. The effort to address these difficulties must be carried on in a dual way: with the collection of new data and with the development of new theoretical frameworks to interpret these data.

In the era of wide-field surveys, dealing with exponentially growing numbers of stars has become a challenge both for observational analyses and for their theoretical interpretation. In this contribution, we will address the latter. The difficulties of dealing with a large number of stars have influenced historically both the classical stellar dynamics and the classic stellar population theories. In classical stellar dynamics, from the few-body problem the attention moved to the mathematical formulation of a many-body problem starting from the pioneering works of Eddington, Chandrasekhar, and others who applied the concepts of statistical mechanics (e.g., the Liouville and Boltzmann equation) and the theory of the potential to "groups of stars" subject to a shared gravitational potential and hence described by a distribution function (e.g., Heggie and Hut, 2003; Saslaw, 1985). In the second half of the past century, a similar concept of "stellar populations" was used initially to address the fundamental equation of stellar statistics, the star-count equation (e.g., Seeliger, 1898; Trumpler and Weaver, 1953). This concept reached the astronomy research field thanks to the observer W. Baade and finally proliferated in the Galaxy modeling field in the 80s (see, e.g., Bahcall and Soneira, 1980; 1984; Bahcall, 1984; Ratnatunga and Bahcall, 1985). In these works, the idea of stellar population involves the photometry alone without phase-space treatment (e.g., Gunn et al., 1981; Tinsley, 1972; 1973). The first works attempting a global model generalization can be dated back to Bienayme et al. (1987), Casertano et al. (1990) and Mendez and van Altena (1996).

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We want to merge these two concepts of stellar populations coming from classical stellar dynamics theory and classical stellar population theory, with the goal to precisely define the minimum condition under which these theories give consistent results. On the one hand, classical stellar dynamics defines a composite stellar population by its density profiles: its natural environment is the phase-space where position and momentum determine the distribution of the stars in the phase-space. On the other hand, classical stellar population theory defines a composite stellar population through its star formation history and initial mass function: its natural environment is the mass and metallicity space within which the stars move according to the fuel consumption theorem. To formulate a comprehensive framework able to account for both the theories is a difficult mathematical task. Here we limit ourselves to the investigation of a simpler, but no less important, task that tightly connects to the star-count modeling techniques. We cast the problem in the following way: if both classical stellar dynamics and classical stellar population theories determine the total mass of a composite stellar population, which is the condition for these approaches to coincide? While for one composite stellar population the answer is known, this is not true for two or more stellar populations. In this work we will derive it for the first time (see also Pasetto et al., 2018a).

In the literature, the concept of multiple stellar populations has long tradition and it is extensively used to study a large variety of topics (e.g., Tosi et al., 1991; Aparicio and Gallart, 1995; Aparicio et al., 1996; 1997; Bertelli and Nasi, 2001; Bournaud and Combes, 2002; Bertelli et al., 2003; Gallart et al., 2005; Vallenari et al., 2006; Bertelli et al., 2008; Tolstoy et al., 2009; Tantalo et al., 2010; Milone et al., 2012; Cubarsi, 2014a; 2014b) even if it still poorly defined or lacking mathematical formalism (see, e.g., Salaris and Cassisi 2006 or Greggio and Renzini 2011 for a review on the subject).

The most remarkable advancement in the mathematical treatment of groups of stars (i.e., populations) probably happened at the beginning of the past century with the introduction of continuous functions: although stars are discrete elements, large gravitationally bound groups of stars sharing common properties started to be studied using continuous distribution functions (DFs) and continuity relations, rather than set-theory (i.e., stars by stars summations). This represented a great advancement with respect to the Celestial mechanics punctual treatment based on the 3-body/few-body problem, etc. In this work we introduce novel mathematical instruments, as the foliations, to address classical stellar population problems.

Pasetto et al. (2012) introduced a new theoretical framework for the concept of stellar populations, and we here endorse and extend it to include multiple composite stellar populations (CSPs). This formalism has the advantage to include in the description of the classical dynamics of a CSP (based on the concept of distribution functions as well) the concepts that are natural to the theory of stellar populations (e.g., initial mass function, star formation rate, etc.). In the treatment that we are proposing, the star birth and death is formally included (hence changing the total number of stars) without any limitation on the nature of their dynamics. The formalism is correct both in the case of a collisional CSP of globular clusters, and a collisionless CSP of a galaxy. Furthermore, this formalism does not depend on the Hamiltonian nature of the dynamics (see Section 4).

The application of this general concept to the Milky Way (MW) has been presented in Pasetto et al. (2016) and will be reviewed briefly in the next section. We start recalling some basic concepts and definitions from Pasetto et al. (2012) and Pasetto et al. (2016) in Section 2.1. In Section 2.2 we set the basis for the idea of multiple stellar populations. In Section 2.3 we have a closer look at the necessary and sufficient condition for a system of the composite stellar population to be coherent in mass. In Section 3 we present two numerical examples which highlight the potential of the theory, in Section 4 we discuss our results and in Section 5 we draw our conclusions. The mathematical aspects of our work are detailed in Appendix A.

2. Theory of multiple composite stellar populations

2.1. Basic concepts of a non-Hamiltonian statistical mechanics for CSPs

A composite stellar population, or simply CSP, is a set of stars born at a different time t, positions $\boldsymbol{x}$, with different velocities $\boldsymbol{v}$, masses M, and chemical compositions Z. We assume that every star lives in the space $\mathbb{E} = M \times Z \times \boldsymbol{\Gamma}$ with $M \subset \mathbb{R}_0^+$ masses, $Z \subset \mathbb{R}_0^+$ metallicity, and $\boldsymbol{\Gamma} \equiv \{\boldsymbol{x}^1, \boldsymbol{v}^1, ..., \boldsymbol{x}^N, \boldsymbol{v}^N\} \subset \mathbb{R}^{6N}$ phase-space (N being the number of stars, and $\mathbb{R}_0^+$ the set of positive real numbers including zero)([1]). At each time t, a single realization of a CSP can be defined as a the s^{th} set of points $E_s \in \mathbb{E}$ defined by some arbitrary properties (i.e., the variable of state of the CSP). Following classical statistical mechanics arguments, we consider not such a single realization of a CSP (microstate), but an infinite collection of the CSPs characterized by the same macroscopic state average (e.g., energy, density, velocity dispersion, metallicity, etc.) but different microscopic conditions, i.e., different microstates s. If a point E_s is representative of the s^{th}-microstate we consider the set of all the $\{s, q\}$: $E_s \neq E_q$ at any arbitrary t. Because the ensemble contains an infinite number of states, the change of the state variables of each CSP happens smoothly, i.e., continuously passing between neighboring states. This allows us to describe the CSPs by a distribution function $f_c : \mathbb{E} \to I \subset \mathbb{R}_0^+$ with I finite interval of the real positive line including zero. Under this hypothesis, the evolution of f_c is given by the Liouville equation for non-Hamiltonian systems (e.g., Colin, 1998) that we write as:

$$\partial_t (g^{1/2} f) + \langle \nabla_{\mathbf{x}}, \partial_t \mathbf{x} g^{1/2} f \rangle = 0, \tag{1}$$

with $g(\mathbf{x}; t)$ being the metric tensor of $\mathbb{E}$ introduced above, which is the classic Liouville equation generalized to (non-Euclidean) dissipative spaces, as we assumed $\mathbb{E}$ to be. Hereafter $\nabla_{\mathbf{x}}$ refers to the gradient over a set of basis coordinates $\mathbf{x}$, $\langle \bullet, \bullet \rangle$ to the inner product, and ∂_t to the partial derivative with respect to the time.[2]

Every time a system presents irreversibility, e.g., the system presents dissipative processes, gas-processes, friction, interaction, merges, etc. it is non-Hamiltonian and non-Hamiltonian statistics has to be used to describe its irreversible dynamics. We can express the Eq. (1) by introducing the evolution operator $\iota \mathcal{E}[\bullet]$:

$$\iota \partial_t f_c = \mathcal{E}[f_c], \tag{2}$$

with ι complex unit, $f_c \equiv \sqrt{g} f$ and $\frac{d}{dt}[\bullet]$ the total derivative operator.[3] As mentioned above, in general the CSPs are non-Hamiltonian entities, and their total number of stars is not conserved. The only hypothesis that we require for Eq. (2) is that the DF is sufficiently smooth so that the necessary derivatives exist; we will assume for simplicity that $f_c \in C^{\infty}(\mathbb{E})$ (i.e., the set of continuous functions with infinitely continuous derivatives). The formal solution of Eq. (2) is then

$$f_c(E; t) = e^{-\iota \mathcal{E} t}[f_c(E; 0)] = e^{-\iota(\mathcal{L} + \Lambda + \mathcal{F})t}[f_c(E; 0)], \tag{3}$$

where, mutating the name from quantum mechanics, we call $e^{-\iota \mathcal{E} t}[\bullet]$ the evolution *propagator*. Here we can decoupled the operator $\mathcal{E}[\bullet]$ linearly, in such a way that $\mathcal{E}[\bullet]$ is split in a part granting the evolution and normalization of f_c given by (standard Liouville operator), and in a part accounting for the compressibility of $\mathbb{E}$ in the case of external fields, say

[1] The choice of the domain of existence is arbitrary and made to exploit the following formalism. Other powerful solutions as $M \subset (\mathbb{R}_0^+)^N$ for the space of masses, $[Fe/H] \subset \mathbb{R}^N$ for the space of metallicity, and $\boldsymbol{\Gamma} \subset \mathbb{R}^{6N}$ for the phase-space, can lead to a formalism in $\mathbb{E} \subseteq (\mathbb{R}_0^+)^N \times \mathbb{R}^N \times \mathbb{R}^{6N}$ that is potentially interesting but more distant from classical stellar population theory.

[2] All these quantities exists because E is assumed to be a Riemannian manifold.

[3] The purpose of the multiplication by the imaginary constant is clearly to obtain an equation similar to the Schrödinger equation, $\iota\hbar\partial_t\psi = H[\psi]$, with $H[\bullet]$ the Hamiltonian operator, $2\pi\hbar$ Plank's constant, ψ wave function, and to work with Hermitian operators (i.e., with real eigenvalues operators) even though we will not exploit here this features of $\mathcal{E}$.