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Kaluza–Klein string cosmological model in $f(R, T)$ theory of gravity

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ABSTRACT

In this paper we have studied Kaluza–Klein string cosmological model within the framework of $f(R, T)$ theory of gravity, where R is the Ricci scalar and T is the trace of the stress energy momentum tensor. We have obtained the solution of the corresponding field equations by using a time varying deceleration parameter. We also discussed various physical and dynamical properties of the model. The variation of different cosmological parameters are shown graphically for specific values of the parameters of the model.

1. Introduction

Einstein's general theory of relativity (GR) explains large number of gravitational phenomena such as bending of light, motion of planet Mercury, expansion of the Universe, etc. GR also predicts the presence of gravitational waves which have been recently detected by international collaboration LIGO. The behavior of solar system like the elliptical orbits of planets/comets/moons around the Sun, their periodicity was possible to explain using the simplest Newtonian mechanics but the complex behavior was possible only after the advancement in the theory of general relativity. Despite these success of GR there are few drawbacks e.g. it fails to explain the accelerated expansion of the Universe. The late time acceleration is usually associated with a strong negative pressure in the form of exotic (and mostly unknown) dark energy (DE). There are many candidates of DE such as the cosmological constant (may be time varying), quintessence, phantom, quantum, etc. which are not explained anywhere in the GR (Utiyama and DeWitt, 1962; Capozziello et al., 2009). At present, it is considered that the Universe is mostly dictated by the presence of 68.3% dark energy, 26.8% dark matter and 4.9% of baryonic matter. In order to study the DE and the consequent cosmic acceleration, several modified theories of gravity have been evolved like, $f(R)$ gravity, $f(R, T)$ gravity, $f(T)$, $f(G)$, $f(R, G)$ gravity, etc. Among these theories, in our study, we considered $f(R, T)$ gravity.

$f(R, T)$ theory of gravity is developed by Harko et al. (2011), where the gravitational Lagrangian is a function of R and trace T of the energy momentum tensor. The dependence on T may be induced by exotic imperfect fluid and/or certain quantum effects. These models also depend on the variation of the matter stress energy tensor (Ram and Priyanka, 2014). $f(R, T)$ theory made advancement in GR due to the coupling of the matter and geometry. Here, the covariant divergence of the stress energy tensor is non–zero. As a result, the motion of test particles is not along geodesic path. The late time cosmic accelerated expansion of the universe can be explain by $f(R, T)$ gravity models. The field equations of $f(R, T)$ gravity can be determined by varying the action of the gravitational field equations with respect to the metric tensor. Several authors e.g. Myrzakulov (2012), Reddy et al. (2012), Chaubey and Shukla (2013), Ram and Kumari (2014), Pawar and Agrawal (2015, 2016), Chirde and Shekh (2015), Agrawal and Pawar (2017), Agrawal and Pawar (2017), etc. have studied various cosmological models in the framework of $f(R, T)$ gravity theory. Samanta (2013) obtained the exact solutions of Kantowski–Sachs cosmological model filled with perfect fluid matter in the presence of $f(R, T)$ gravity. Mishra and Sahoo (2014) derived Bianchi type VI_h cosmological model filled with perfect fluid in $f(R, T)$ gravity. Moraes and Sahoo (2017) have discussed the modeling of static wormholes in the framework of $f(R, T)$ gravity.

$f(R, T)$ gravity is also useful in string cosmology models. The study of string cosmological models (as cosmic strings) find considerable attention in cosmology. It is assumed that cosmic strings play pivotal role in the early evolution of the universe, particularly before the particle creation. Grand unified theories anticipate such strings to be formed during the phase transition after the big–bang explosion at the temperature less than the critical temperature. Furthermore, it is predicted that cosmic strings are linear topological defects associated with spontaneous symmetry breaking whose plausible production site is cosmological phase transitions in the early universe. They also considered to be acting as a gravitational lens and therefore are assumed as possible seeds for the formation of galaxies. Letelier (1979), Stachel (1980), Vilenkin (1981), Letelier (1983) and Gott (1985) have broadly discussed the gravitational effects of cosmic strings in general relativity. Krori et al. (1990) and Tikekar and Patel (1992) have

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examined Bianchi type space–time for a cloud string. Raj and Anirudh (2007) have determined Bianchi type–*III* string cosmological models in the presence of bulk viscous fluid for massive string. Pawar et al. (2008) discussed dust magnetized string cosmological model while Rao et al. (2008) investigated the string cosmological model in Saez–Ballester theory of gravitation. Sahoo et al. (2016) have discussed Bianchi–*III* and –VI_0 string fluid source cosmological models in the framework of *f*(*R, T*) gravity. Beside this, Pawar and Deshmukh (2010) have elaborated plane symmetric string cosmological model with bulk viscosity and Pawar and Patil (2012) have discussed the plane symmetric cosmological models with string dust magnetized bulk viscosity in Lyra geometry. Sahoo and Mishra (2013) studied the plane symmetric cosmological solutions for quark matter with the string cloud and domain walls using Rosenâs bimetric theory. Chodos and Detweiler (1980), Yoshimura (1984) and Chatterjee (1993) have studied higher dimensional string cosmological models in the context of different theories.

Generally, with the usual (four dimensional) space–time, the consolidation of gravitational forces with other forces of nature is not possible. Thus to improve the possibility of geometrically unifying the fundamental interactions of the universe, study of higher dimensional space–time is essential. In particle physics, various experiments were initiated to develop the higher dimensional cosmological models that resulted into the formulation of the Kaluza–Klein theory. Kaluza–Klein (KK) theories reveal how the gravity and the electromagnetism can be unified from Einstein's field equations generalized to five dimensions. Subsequently, different authors studied physics of the universe in the context of higher dimensional space–time viz. Alvarez and Gavela (1983), Randjbar-Daemi et al. (1984) and Marciano (1984) have shown that the experimental detection of time variation of fundamental constants could provide evidence for extra space dimensions. Moreover, multi–dimensional cosmological models are studied by several authors in the framework of different theories (Chodos and Detweiler, 1980; Lorenz-Petzold, 1985; Ibáez and Verdaguer, 1986; Gleiser and Diaz, 1988; Reddy and Venkateswara Rao, 2001). Jain et al. (2013) have obtained exact solutions of Einstein field equations of Kaluza–Klein cosmological model with strange quark matter and string cloud. Pawar and Pawar (2015) have studied Kaluza–Klein cosmological model in the presence of *f*(*R, T*) theory of gravity. Katore and Hatkar (2015) have determined Kaluza–Klein universe in the presence of magnetized dark energy in the reference of Lyra manifold.

Motivated by the above works in the cosmology, here we present the study of Kaluza–Klein string cosmological model in the *f*(*R, T*) theory of gravity. In Section 4, we obtained the solutions of the field equations by considering a power–law relation between the scale factors and the special law of variation of Hubble's parameter proposed by Berman (1983). In the subsequent section, we derived few physical parameters of the model. In the last section, we discuss the physical behavior of the model with the help of the derived physical parameters.

2. Gravitational field equations of f(R,T) gravity

Harko et al. (2011) has obtained the field equations of *f*(*R, T*) theory of gravity from the Hilbert-Einstein variational principle by considering the metric-dependent Lagrangian density L_m. The action for *f*(*R, T*) gravity is given as,

$$S = \int \frac{1}{16\pi} f(R, T)\sqrt{-g}\,d^4x + \int L_m\sqrt{-g}\,d^4x, \tag{1}$$

where L_m is the matter Lagrangian density and *f*(*R, T*) is function of Ricci scalar *R* and *T*. *T* is the trace of energy momentum tensor of the matter T_{ij} which is given by,

$$T_{ij} = \frac{-2\partial(\sqrt{-g}L_m)}{\sqrt{-g}\,\partial g^{ij}}, \tag{2}$$

and the trace of energy momentum tensor is $T = g^{ij}T_{ij}$. Here, it is considered that the matter Lagrangian L_m depends on the metric tensor component g_{ij} rather than its derivatives. Which implies,

$$T_{ij} = g_{ij}L_m - \frac{\partial L_m}{\partial g^{ij}}. \tag{3}$$

The field equations of *f*(*R, T*) gravity are given by varying the action *S* with respect to metric tensor g_{ij}

$$f_R(R, T)R_{ij} - \frac{1}{2}f(R, T)g_{ij} + (g_{ij}\Box - \nabla_i\nabla_j)f_R(R, T) = 8\pi T_{ij} - f_T(R, T)T_{ij} - f_T(R, T)\Theta_{ij}, \tag{4}$$

where the value of Θ_{ij} is,

$$\Theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{l\alpha}\frac{\partial^2 L_m}{\partial g^{ij}\partial g^{l\alpha}}. \tag{5}$$

Here $f_R(R, T) = \frac{\partial f(R,T)}{\partial R}$, $f_T(R, T) = \frac{\partial f(R,T)}{\partial T}$, $\Box = \nabla^n\nabla_n$ where ∇_n is the covariant derivative.

By contracting Eq. (4), we obtain

$$f_R(R, T)R + 3\Box f_R(R, T) - 2f(R, T) = 8\pi T - f_T(R, T)(T + \theta), \tag{6}$$

where $\Theta = g^{ij}\Theta_{ij}$.

The stress energy tensor of the matter is obtained by using matter Lagrangian L_m as,

$$T_{ij} = (\rho + p)u_iu_j + pg_{ij} - \lambda x_ix_j, \tag{7}$$

where ρ is the energy density of the fluid and *p* is the pressure of the fluid. Here, $u^i = (0, 0, 0, 0, 1)$ is the five-velocity vector in co–moving co–ordinate system and satisfies the condition $u_iu_j = -x_ix_j = -1$, $u_ix_i = 0$ where is the direction of the string and $u^i\nabla_ju_i = 0$. We choose a perfect fluid matter characterized by $L_m = -p$ to obtain

$$\Theta_{ij} = -2T_{ij} - pg_{ij}. \tag{8}$$

The field equations of *f*(*R, T*) gravity depends on the tensor Θ_{ij}. Thus depending on the nature of the matter source, various cosmological models of the *f*(*R, T*) gravity are possible. Initially, Harko et al. (2011) derived three classes of models using following functional forms of *f*,

$$f(R, T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)f_3(T). \end{cases} \tag{9}$$

Harko argued that, the field equations depend on the physical nature of the matter field. Hence depending on the nature of the matter source, for each choice of *f* we can obtain several theoretical models, corresponding to different matter models. Among these, we considered second class, i. e. $f(R, T) = f_1(R) + f_2(T)$ with $f(T) = \mu T$ and $f(R) = \mu R$, where *f*(*T*) is the arbitrary function of stress energy of matter, *f*(*R*) is the arbitrary function of the Ricci scalar *R* and μ is an arbitrary constant. We choose this simple form as it becomes *f*(*R*) if $f_2(T) = 0$ and also it makes calculations more simpler. Eq. (4) gives the gravitational field equations of *f*(*R, T*) gravity as follows,

$$R_{ij} - \frac{1}{2}Rg_{ij} = \left(\frac{8\pi + \mu}{\mu}\right)T_{ij} + \left(p + \frac{1}{2}T\right)g_{ij}. \tag{10}$$

3. Metric and the field equations

We consider the Kaluza–Klein metric as