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Acoustic geometry obtained through the perturbation of the Bernoulli's constant

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A B S T R A C T

For accretion onto astrophysical black holes, we demonstrate that linear perturbation of Bernoulli's constant defined for an inviscid irrotational adiabatic flow of perfect ideal fluid gives rise to phenomena of analogue gravity. The formulation of our work is done in the Newtonian framework and also within the General relativistic framework, i.e., considering a static spacetime background, as well. The resulting structure of the analogue acoustic metric is similar to the acoustic metric found in perturbing velocity potential and mass accretion rate.

1. Introduction

Except for the supersonic stellar wind fed accretion (Lamers and Cassinelli, 1999; Frank et al., 1985; Kato et al., 2008), accretion flows onto astrophysical black holes are necessarily supersonic (Liang and Thompson, 1980). For low angular momentum accretion with practically constant specific angular momentum, more than one sonic points may form in such flow and a stationary shock may join two such transonic solutions passing through two such sonic points (Das and Czerny, 2012; Liang and Thompson, 1980; Abramowicz and Zurek, 1981; Muchotrzeb and Paczynski, 1982; Muchotrzeb, 1983; Fukue, 1983; 1987; Lu, 1985; 1986; Muchotrzeb-Czerny , 1986; Abramowicz and Kato, 1989; Abramowicz and Chakrabarti, 1990; Chakrabarti, 1996; Kafatos and Yang, 1994; Yang and Kafatos, 1995; Pariev, 1996; Peitz and Appl, 1997; Caditz and Tsuruta, 1998; Das, 2002; Das et al., 2003; Barai et al., 2004; 2004; Abraham et al., 2006; Das et al., 2007). The formation of such shocks can be explained through time dependent numerical simulation works (Okuda et al., 2004; 2007; Suková and Janiuk, 2015b; 2015a; Suková et al., 2017). Such post shock flow can manifest its properties through the characteristic black hole spectra and can help to understand the observational signature of the astrophysical black holes in the universe (Das et al. (2015); Mocibrodzka et al. (2006) and references therein). Such shocked multi-transonic flows are essentially barotropic, inviscid, irrotational transonic fluid flow under the influence of the strong gravitational field in presence of gravitational (black hole) event horizon.

It is, however, to be noted that the characteristic black hole spectra for transonic accretion are usually computed for steady state flow, considering that such steady state is stationary. Time variability and various kind of fluctuations are, however, not very uncommon in large scale astrophysical flows. It is thus imperative to ensure that the steady state integral transonic accretion solutions are stable under perturbation.

In recent years, much attention have been paid to study the analogue gravity phenomena in classical (non quantum), where for a supersonic irrotational inviscid flow governed by a barotropic equation of state, the propagation of the linear acoustic perturbation (sound wave) within that fluid can be described by acoustic metric and a sonic spacetime can be formed embedded within such stationary background fluid flow (Unruh (1981); Visser (1998); Barcelo et al. (2005); Novello et al. (2002)). Such sonic geometry contains an acoustic horizon from where Hawking like radiation may be produced.

Study of such sonic geometries embedded within the transonic accretion flow can thus be very important to investigate certain novel features of such phenomena. Accreting black holes is the only system found in the universe where both type of horizons, gravitational as well as acoustic, can be formed, and the same fluid can pass through both type of horizons as well. Hence theoretically if one would like to compare the properties of these two types of horizons, accreting black holes may be considered as the best candidate to study the sonic geometry embedded within it. Also, in usual analogue models, the gravitational field does not play any role while formulating the corresponding sonic geometry. For accretion onto astrophysical black holes (for accretion onto any compact massive astrophysical objects in general), the gravity determines the dynamics of the fluid and hence the associated acoustic spacetime itself is influenced by the gravitational field.

For purely classical analogue systems, the detailed analysis of the

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quantum Hawking like effects may not always be possible to study, however, the study of the acoustic surface gravity can have deep significance in such systems. The acoustic surface gravity itself is a rather crucial entity to understand the flow structure as well as the associated sonic metric, and can thus be studied independently without looking into the existence of any analogue radiation (of phonons) like thermal phenomena characterized by their very feeble temperature too impractical to detect through any present day experimental set up. In recent years, the role of the analogue surface gravity in studying the non negligible effects associated with the emergence of the stimulated Hawking effects has been highlighted by examining such effects through the modified dispersion relations. Such study have been performed from the purely analytical point of view as well as within the experimental set up (Rousseaux et al., 2008; 2010; Jannes et al., 2011; Weinfurtner et al., 2011; Leonhardt and Robertson, 2012; Robertson, 2012).

The deviation of the Hawking like effects within a dispersive media, i.e., within the fluid under consideration, from the universal behaviour of the original Hawking effect, depends sensitively on the gradient of the background bulk stationary velocity, as it has recently been suggested. It is, however, important to note that such theory of the non universal feature of the Hawking radiation has been postulated essentially for the isothermal flow and hence the space gradient for the sound velocity has not been taken into account. Also, the exact numerical values corresponding to the velocity gradient has not been possible to found yet and has been approximated by making certain assumptions.

For stationary integral accretion solutions as discussed in aforementioned paragraphs, the values of the space gradient of *both* the dynamical flow velocity as well as the speed of propagation of the acoustic perturbation have been computed very accurately using numerical schemes. Hence it is obvious that if the accreting black hole system can be studied as a classical analogue system, the non universal features of the Hawking like effects can further be modified including certain novel features.

It is thus obvious that the accreting black hole systems, although may not provide any direct signature of the Hawking like temperature (analogue temperature arising out from the phonon radiation) can still be considered as a very important as well as a unique theoretical construct to study the analogue gravity effects.

The study of the stability properties of the background integral accretion solutions can lead to the emergence of the analogue phenomena which we shall demonstrate in the present work. We shall linear perturb various accretion related physical variables and will show that such perturbation does not grow indefinitely and hence the steady state accretion flow is actually stable under linear perturbation. We shall not perform any non linear stability analysis in the present work. We shall also show that such perturbation analysis leads to the formation of the acoustic metric embedded within the fluid flow.

In this connection, it is to be mentioned that one can make attempt to study the analogue phenomena for the primordial micro black holes as well for which the analogue as well as the original Hawking temperature will be significantly large and will perhaps be a measurable quantity. It is, however, to be noted that the kind of analogue system we are interested in, can be emerged automatically in presence of accretion only. The theory of accretion flows onto primordial black hole are not a very well understood phenomenon as of now. To study the analogue effects for such black hole systems, one first has to formulate a self consistent theory of accretion processes around micro black holes, which, in our opinion, is a rather involved task to accomplish, and is obviously beyond the scope of the present work. Hence in the present work, we concentrate on large astrophysical black holes to study the analogue effects.

Mass accretion rate is a quantity having a reasonable physical significance in accretion phenomena. Linear perturbation of mass accretion rate in sub-Keplerian disk accretion in non relativistic framework also behaves like a massless scalar field in curved space-time (Nag et al., 2012), i.e., analogue gravity also emerges when accretion rate is perturbed.

Several works have been done in general relativistic framework as well. Linear perturbation of velocity potential in curved space-time background shows analogue gravity effect (Bilic, 1999). Similarly, linear perturbation of mass accretion rate in accretion of perfection fluid in curved space-time background also shows same effect (Ananda et al., 2015; Bollimpalli et al., 2017; Bilic, 1999).

In this work, we've shown that linear perturbation of another quantity, the Bernoulli's constant which is the integral solution of the corresponding Euler equation, also produces similar acoustic geometry. The whole work is being done in the non-relativistic framework and relativistic framework as well. Accretion phenomena of adiabatic flow are chosen to illustrate the fact. Radial accretion having spherical symmetry as well as disk accretion having axial symmetry are considered.

The linear perturbation technique also has astrophysical significance. We get a wave equation of the linear perturbation of Bernoulli's constant which is similar to the massless scalar field Klein–Gordon equation in curved space-time geometry. The nature of the solution of this wave equation tells us whether the existing steady state solutions like steady state solution for Bondi accretion, are stable or not under such perturbation in the medium. We have done stability analysis for that and we have concluded that not only the integrals of motion play a crucial role to determine the dynamics of the accretion flow in steady state but also their linear perturbations govern the behaviour of all the dynamical and thermodynamic quantities in the time dependent problem within the perturbative framework.

In our work, the correspondence between a classical (non-quantum) analogue model and the accretion processes onto astrophysical black holes has been established through the process of linear stability analysis of stationary integral transonic accretion solutions corresponding to the steady state flow only. That means, only such accreting black hole systems have been considered which are in steady state. The body of literature in accretion astrophysics, however, is huge and diverse. There are several steady state flow models which may not be multi-transonic, and there are several excellent works which deal with non steady hydrodynamic accretion (which may not contain multiple sonic points or shocks) which may include various kind of time variabilities and instabilities, using complete time-dependent numerical simulation (Hawley et al. (1984a,b); Kheyfets et al. (1990); Hawley (1991); Yokosawa (1995); Igumenshchev et al. (1996); Igumenshchev and Beloborodov (1997); Nobuta and Hanawa (1999); Molteni et al. (1999); Stone et al. (1999); Caunt and Korpi (2001); De Villiers and Hawley (2002); Proga and Begelman (2003); Gerardi et al. (2005); Moscibrodzka and Proga (2008); Nagakura and Yamada (2008, 2009); Janiuk et al. (2009); Bambi and Yoshida (2010b,a); Barai et al. (2011, 2012); Suková and Janiuk (2015a); Zhu et al. (2015); Narayan et al. (2016); Mościbrodzka et al. (2016); Suková et al. (2017); Sądowski et al. (2017); Mach et al. (2018); Karkowski et al. (2018); Inayoshi et al. (2018); Fragile et al. (2018)). We, however, did not concentrate on such approach. In the present paper, our main motivation was to explore how the analogue gravity phenomena can be addressed through the linear stability analysis of steady-state solutions of hydrodynamic accretion.

2. Acoustic gravity in non-relativistic framework

In non-relativistic frame work, fluid velocity is much less than light speed. The momentum conservation equations and mass conservation equation for fluid is taken (Clarke and Carswell, 2007) according to Newton's laws of dynamics. The continuity equation of fluid is given by

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}.(\rho\vec{v}) = 0 \tag{1}$$

where $\rho, \vec{v}$ are fluid density and velocity respectively. Euler