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Effects of mass variation on structures of differentially rotating polytropic stars

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ABSTRACT

A method is proposed for determining equilibrium structures and various physical parameters of differentially rotating polytropic models of stars, taking into account the effect of mass variation inside the star and on its equipotential surfaces. The law of differential rotation has been assumed to be the form of $\omega^2(s) = b_1 + b_2 s^2 + b_3 s^4$. The proposed method utilizes the averaging approach of Kippenhahn and Thomas and concepts of Roche-equipotential to incorporate the effects of differential rotation on the equilibrium structures of polytropic stellar models. Mathematical expressions of determining the equipotential surfaces, volume, surface area and other physical parameters are also obtained under the effects of mass variation inside the stars. Some significant conclusions are also drawn.

1. Introduction

Theoretical model of a star is essentially a self-gravitating gaseous sphere in hydrostatic and thermal equilibriums. Theoretical studies of the problem of the equilibrium structure of a gaseous sphere are often carried out to understand the nature of internal structures responsible for various observed phenomena of the stars. Some of the stars are observed as single while others are observed in groups of two or more. Observations have shown that some of the stars are rotating about their axes of rotation. This rotation may be a solid body rotation or a differential rotation. Many of the stars in binary and multiple systems are also known to be rotating about their axes as well as revolving around each other. It may be assumed that the model of a single non rotating star is a simple gaseous sphere while the equilibrium model of a rotating star will be rotationally distorted gaseous sphere. Similarly, the equilibrium model of a star appearing in a binary or a multiple system will be a tidally distorted gaseous sphere if it is not rotating and a rotationally and tidally distorted gaseous sphere if is rotating as well. Rotational forces are also expected to influence the inner structure and dynamical stability of such stars.

Chandrasekhar and Lebovitz (1962) developed a theory of distorted polytropes. Since then several authors such as Kopal (1968), Lal et al. (2006) have addressed themselves to these problems. In some approximations such as given by Mohan and Singh (1978), Saini et al. (2012), Mohan et al. (1990, 1992, 1994), actual equipotential surfaces of a rotationally and tidally distorted star are approximated by equivalent rotationally and tidally distorted Roche equipotentials. Lal et al. (2006) have applied this approach on polytropic stars and hence found their equilibrium structures. In this approximation, Kippenhahn and Thomas (1970) averaging approach and results of the Roche equipotentials obtained by Kopal (1968), are used to incorporate the rotational and tidal effects up to second order of smallness for modelling of stellar structure.

However, mathematical problem to determine the effects of rotation on the equilibrium structures and stabilities of realistic models of stars are quite complex. This study becomes more complex if the density of the stars is not uniform from centre to surface.

In this paper, some physical parameters have been obtained in relation to the structures of differentially rotating gaseous spheres, using the law of differential rotation of the form:

$$\omega^2(s) = b_1 + b_2 s^2 + b_3 s^4, \quad (1)$$

where $\omega(s)$ being the angular velocity of rotation of a fluid element at distance s from the axis of rotation while b_1, b_2 and b_3 are numerical constants as mentioned in Table 1.

Our technique utilizes the averaging approach of Kippenhahn and Thomas (1970) and concepts of Roche-equipotential in a manner earlier used by Saini et al. (2012) to incorporate the effect of differential rotation on the rotationally distorted stellar models. Inner structures and various physical parameters of differentially rotating polytropic models of polytropic indices 1.5, 2.0, 3.0 and 4.0, have been computed with suitable combination of the parameters b_1, b_2 and b_3. This law of

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Table 1
Values of the parameters for using in the law of differential rotation.

Models →		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Parameters	b_1	0.0	0.1	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.04	0.1	0.1	0.04
	b_2	0.0	0.0	0.1	0.0	0.1	0.1	0.2	−0.05	−0.1	−0.15	−0.02	−0.01	0.02	−0.1	−0.16
	b_3	0.0	0.0	0.0	0.1	0.0	0.1	0.1	0.0	0.05	0.1	0.4	0.0625	−0.05	0.0	0.16

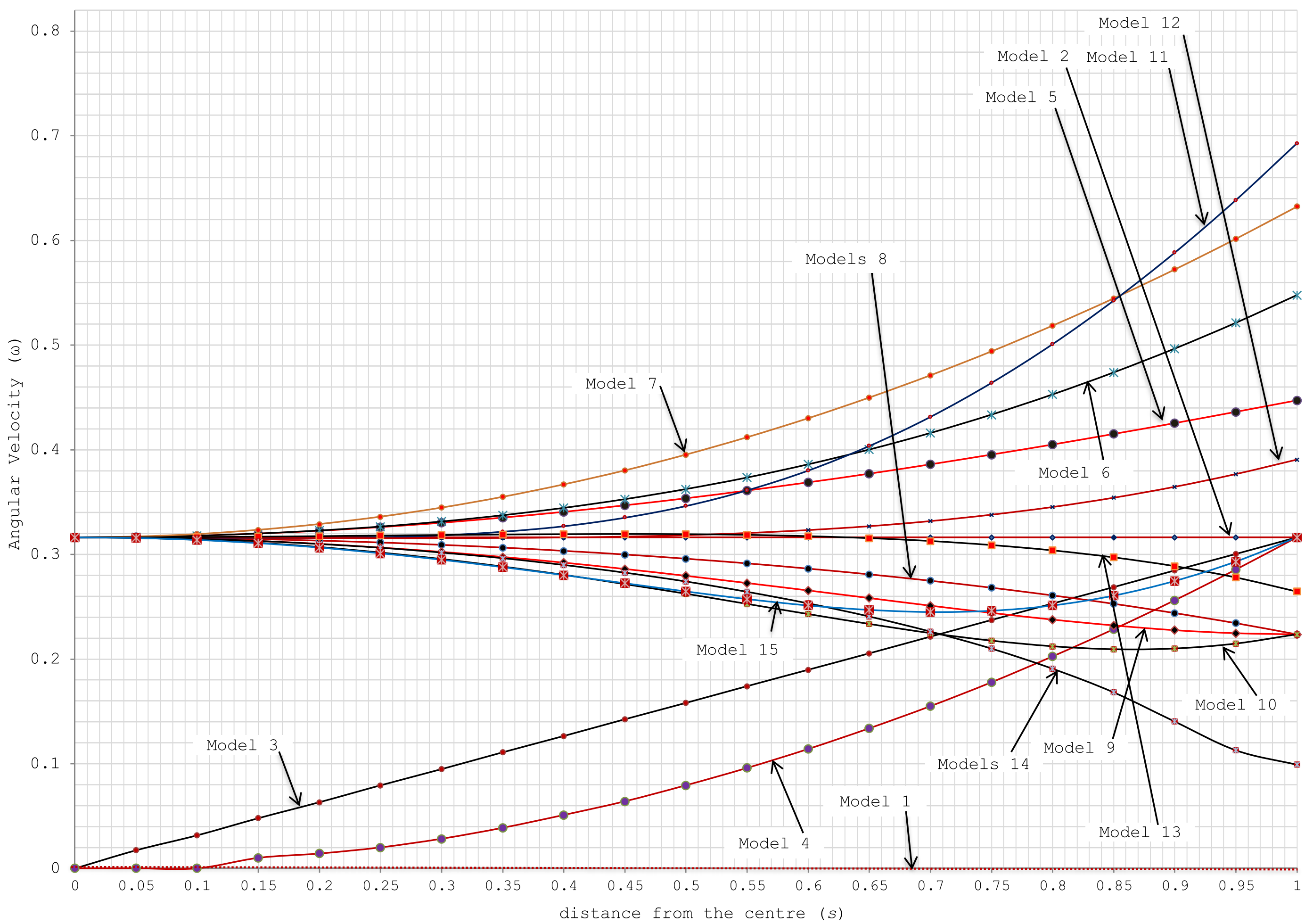

Fig. 1. Angular velocities of stellar models versus ‘s’.

differential rotation was earlier used by Mohan et al. (1992) to obtained equilibrium structures of the stars. Mohan et al. (1992) have also checked dynamical stabilities of assumed stellar models (mentioned in Table 1), obeying this law, by using stability criterion given by Stoeckly (1965). According to this criterion, a differentially rotating stellar model, rotating according to the law $\omega = \omega(s)$, is stable if:

$$\frac{d}{ds}\{s^2.\ \omega(s)\} > 0. \quad (2)$$

All models (Table 1) are found stable. The behaviours of angular velocities of these assumed models versus distance s have been given in Fig. 1.

The general problem of determining the equilibrium structures of a class of differentially rotating polytropes models of stars has also been investigated. This is a general theoretical study and can be used for stars of any desired mass in any evolutionary phase. Whereas polytropic index 3 is generally considered an appropriate index for the main-sequence stars. It is 1.5 for pre-main sequence stars and 4 for giants.

2. Polytropic models of stars

For a polytropic model, pressure P and density ρ at any arbitrary point inside the star, are given by the relations:

$$P = P_C\theta^{N+1} \text{ and } \rho = \rho_C\theta^N, \quad (3)$$

where P_c and ρ_c are the values of pressure and density at the centre, respectively. θ is the parameter depending upon the distance of the chosen point from the centre, such as $0 \le \theta \le 1$. The index N used in relation (3), is called polytropic index of the stellar model, measures the central condensation of stellar models. Generally, the value of N lies between zero to five. Polytropic model of index zero has a homogeneous structure, in which density is uniformed throughout the model, while a polytropic model of index five is highly centrally condensed model, whose radius extends to infinity.

The equilibrium structure of a polytropic model of index N is determined by the solution of the non-linear differential equation: