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The Hall-induced stability of gravitating fluids

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ABSTRACT

We analyze the stability behavior of low-density partially ionized self-gravitating magnetized unbounded dusty plasma fluid in the presence of the Hall diffusion effects (HDEs) in the non-ideal magnetohydrodynamic (MHD) equilibrium framework. The effects of inhomogeneous self-gravity are methodically included in the basic model tapestry. Application of the Fourier plane-wave perturbative treatment decouples the structuration representative parameters into a linear generalized dispersion relation (sextic) in a judicious mean-fluid approximation. The dispersion analysis shows that the normal mode, termed as the gravito-magneto-acoustic (GMA) mode, is drastically modified due to the HDEs. This mode is highly dispersive, and driven unstable by the Hall current resulting from the symmetry-breaking of electrons and ions relative to the magnetic field. The mode feature, which is derived from a modified induction with the positive Hall, is against the ideal MHD. It is further demonstrated that the HDEs play stabilizing roles by supporting the cloud against gravitational collapse. Provided that the HDEs are concurrently switched off, the collapse occurs on the global spatial scale due to enhanced inward accretion of the gravitating dust constituents. It is seen explicitly that the enhanced dust-charge leads to stabilizing effects. Besides, the Hall-induced fluctuations, as propagatory wave modes, exhibit both normal and anomalous dispersions. The reliability checkup of the entailed results as diverse corollaries and special cases are illustratively discussed in the panoptic light of the earlier paradigmatic predictions available in the literature.

1. Introduction

The interstellar dusty media and their denser regions experienced normally as dust molecular clouds (DMCs), which are well known to be the large-scale star formation sites, support a variety of collective natural wave phenomena (Pudritz, 1990; Lou, 1996; Priest, 2014; Borah and Karmakar, 2015). They are partially ionized plasmas having a very small fraction of ionization ($\sim 10^{-2}$–10^{-6}%), with a widely accepted mean value of degree of ionization as 10^{-5}% (Caselli et al., 1998; Spitzer, 2004; Pandey and Vladimirov, 2007). The ionization, as is well known in the case of HII region, is sourced by different mechanisms, such as photo-ionization, secondary emission, etc. The heavy neutrals (radicals) can capture the released (free) electrons and become charged (multiply ionized), which are conventionally termed as charged dust grains, dust microspheres, or dust particulates (Borah and Karmakar, 2015). The magnetic field is one of the important source ingredients of the interstellar media, widely realizable in the form of particle dynamics. The origin of the magnetic field is still in debate and it is unclear whether it is primordial or galactic in origin (Passot et al., 1995).

It may be understood that the convective flow dynamics of the charged particles in the plasma background produces electric currents, which as a consequence, generate the magnetic field, regardless of its strength. This field plays unique role in dust coagulation mechanism, but no role in dust charging process. It plays an important role in the star formation process by supporting the cloud against the gravitational collapse (Borah and Karmakar, 2015). The support may be static by slowing down the collapse directly, or dynamic via the different types of magnetohydrodynamic (MHD) waves. The waves create an outward stress in the plasma, and prevent it from inward free-fall fragmenting collapse (Dewar, 1970; Pudritz, 1990).

The relative drift among the plasma species or between the plasma and neutral particles gives rise to different types of non-ideal MHD effects (Wardle, 2007; Pandey and Wardle, 2008). It depends on the plasma density and degree of ionization. When both the electrons and ions are frozen with the field, they drift in the sea of neutrals (ambipolar diffusion). In this case, the cyclotron frequencies of both the species are higher than their respective collision frequencies (low-density and high-ionization plasmas). When both the electrons and ions are decoupled from the field, the diffusion turns into the Ohmic one. It occurs when cyclotron frequencies are lower than their corresponding collision counterparts (e.g., high-density and low-ionization plasmas).

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When only the electrons are coupled with the magnetic field, the Hall diffusion effects (HDEs) come into picture (intermediate density-ionization range), where the electron cyclotron frequency dominates over its collisional counterpart, whereas the ion collision frequency prevails over its cyclotron frequency. Thus, the Hall implicates a symmetry breaking of the electrons and ions relative to the considered magnetic field in the complex astrophysical grainy cloud.

It is seen that the HDE arises in both weakly and highly ionized plasmas; but, with unlike mechanisms on different scales of space and time (Pandey and Wardle, 2008; Pandey et al., 2008). As such, when the electrons are frozen in the magnetic field and the ions are dissociated due to collision, a transverse electric field develops that drives the electrons in the sea of ions and neutrals resulting in the HDE (D'Angelo, 2003). It occurs in weakly ionized plasma and exists on large scales, typically on the order of system size. In highly ionized media, where both the electrons and ions are coupled with the magnetic field, the difference in mass gives rise to the relative drift, and hence, the HDE emerges (Pandey and Wardle, 2008). Here, the frequency of magnetic field fluctuations is much higher than the ion cyclotron frequency which disables the ions to follow it. On the other hand, the electron cyclotron frequency being comparable to the magnetic fluctuation frequency, the electrons remain tied with the magnetic field. It exists on the scale of ion skin depth, which in principle is much smaller than the system size.

The Hall MHD operates in a wide range of astrophysical plasmas, such as neutron stars, interstellar and interplanetary media, pre-stellar disks, protostellar disks, upper atmospheres of the Sun, and so forth (Balbus and Terquem, 2001; Pandey and Vladimirov, 2007). It provides the mechanisms for origin and propagation of different types of waves, such as the circularly polarized whistler waves (Balbus and Terquem, 2001), fast and slow ultra-low frequency (ULF) electromagnetic waves (Aburjania et al., 2005), kink and sausage waves (Miteva et al., 2003), and so on. In the upper photosphere and chromosphere of the Sun, it has a significant role to play in the excitation and propagation processes of the waves (Pandey et al., 2008). Apart from active participation in destabilizing the gravitating fluids (Hosking, 1967; Hosking and Dewar, 2016), the HDE can also provide significant information about the turbulence-sustenance in the eruptive protostellar disks (Balbus and Terquem, 2001). The HDE can also enhance other non-ideal MHD effects, especially, the Ohmic dissipation, and causes the decay of the magnetic field lines (Goldreich and Reisenegger, 1992). A significant role of the HDE is also known in controlling the properties (e.g., scale-size, angular momentum transport, etc.) of circumstellar disks around neonatal stellar structures in the like grainy plasma media (Wurstur et al., 2016).

The Hall MHD, as already pointed out, can significantly affect the magnetically mediated star formation processes in the molecular clouds by slowing down dynamical condensation against the collapse (Braiding and Wardle, 2012). In this context, there have been a number of studies in the past showing the effects of the Hall diffusion current on various plasma stability behaviors. Pandey and Vladimirov (2007) have investigated the parametric instability of the Alfven wave with the HDE in the mean-fluid approach. A number of researchers (Pudritz, 1990; Balsara, 1996; Lou, 1996) have studied the Jeans stability in magnetized plasmas with one- and two-fluid approaches; but, with the HDE absolutely ignored. In particular, Pudritz (1990) has shown that the global collapse of the largest possible DMCs can be prevented against self-gravity by the cloud magnetization and its directional tuning. Next, Balsara (1996) has demonstrated that the gravitational collapse can act as a source for the sustenance of turbulence. What is more,Lou (1996) has reported an interesting result that, in the magnetized plasmas, the critical Jeans mass of the cloud increases considerably due to the magnetic field. The magnetized (enhanced) Jeans mass ($M_{J,\,B}$) over the unmagnetized (usual) Jeans mass (M_J) can be expressed as $M_{J,B} = M_J(1 + M_A^2)^{1/2}$; where, $\lambda_{J,B} = 10\lambda_J(1 + M_A^2)^{1/2}$ is the new Jeans length, drastically modified by the Alfvenic Mach number $M_A = v_A/c_S$. In normal clouds (Spitzer, 2004), the above interrelation becomes $M_{J,\,B} \sim 10M_J$, which is a basic parameter indicating the supercritical initiation of dynamical cloud collapse, but, under the condition of the dynamical timescale exceeding the fragmentation timescale (Lou, 1996). Thus, it is clearly seen that the effects of the HDE-induced electric current and grain magnetization on the wave-stability-kinetics leading to star-planet or other bounded structure formation, started by the canonical Jeans fluid dynamical mechanism, is hitherto remaining unexplored and ill-understood to the best of our knowledge.

This paper, motivated and driven mainly by the HDE-induced astrophysical scenarios, aims at such instabilities in the light of the mean-fluid model in the presence of self-gravitational plasma Hall dynamics. We apply the Fourier plane-wave analysis over the basic perturbed non-ideal magnetohydrodynamic (MHD) structuring equations to derive the required polynomial dispersion relation (sextic). It enables us to unveil a modified normal mode existing in the cloud, the *gravito-magneto-acoustic* (GMA) mode, which is nontrivial in dispersion characteristics. Its propagatory characteristics are illustratively discussed. The implications and realistic applications for large-scale bounded structure formation in galaxies are reinforcingly indicated.

We organize the layout of the manuscript in a standard structure as follows. Section 1, as already discussed, gives a brief introduction of the Hall MHD effects in the astrophysical and cosmic environments. Section 2 deals with the physical model and mathematical formalism, expounded in two Sections 2.1 and 2.2, respectively. The main findings, alongside special corollaries and manifestations in the light of existing predictions, are discussed in Section 3. Finally, the main conclusion and future scope are summarily outlined in Section 4.

2. Physical model

We consider a low-density partially ionized self-gravitating magnetized dusty plasma system in multi-fluidic framework. It is a complex plasma composed of the lighter electrons, heavy ions, heavier dust grains (e.g., impurity complexes of silicate and ionic radicals) and neutral particles, collectively treated as a quasi-neutral mean-fluid. Thus, it is, for the fluid (MHD) model approximation (Tsytovich et al., 2014), presumed at the outset, that all the collisional scales (mean free path, the mean time of flight) are much smaller than the characteristic scales of mean- fluid flow (Jeans spatiotemporal scales). In other words, the model considers frequent collisions of the grains with all the plasma constituent particles. In addition, it is assumed that the mean fluctuation velocity ($v_p = \omega/k$) lies within the thermal scaling, with all the usual notations described later, defined as $\sqrt{k_B T_e/m_e}$, $\sqrt{\mathrm{k_B} T_i/m_i} > > \omega/k \geq \sqrt{k_B T_d/m_d}$. The plasma effects on the grain growth dynamics is ignored for simplicity. The neutral gaseous fluid is coupled with the collapsing plasma by frictional mechanism. The inhomogeneities due to fluid turbulence, which might affect the gravitational cloud fragmentation (Shu et al., 1987), are ignored.

The Hall effect normally arises when the electrons are strongly coupled with the magnetic field lines ($\beta_e > \ > 1$, frozen-in-field), and the ions are decoupled ($\beta_i < \ < 1$, frozen-out-field) from the field due to collisional effects, i.e., when the condition $\beta_i < \ < 1 < \ < \beta_e$ is satisfied. The coupling parameter $\beta_j(=\omega_{cj}/\nu_j)$ is the ratio of the cyclotron frequency $\omega_{cj}(=q_j B/m_j c)$ to the collision frequency ν_j. Here, ν_j is contributed concurrently by the charged-neutral and charged-charged collisions. This implies that, for the electrons, the collision frequency is given by, $\nu_e = \nu_{en} + \nu_{ei}$; and for the ions, $\nu_i = \nu_{in} + \nu_{ie}$. The electron-ion collision frequency is different from the ion-electron collision frequency due to their different mobilities (Bellan, 2004). The respective electron-ion and ion-electron collision frequencies can be expressed as, $\nu_{ei} = 4\sqrt{2\pi}\lambda e^4 Z^2 n_i/3\sqrt{m_e} k_B^{3/2} T_e^{3/2}$, and $\nu_{ie} = 4\sqrt{\pi}\lambda e^4 Z^4 n_i/3\sqrt{m_i} k_B^{3/2} T_i^{3/2}$ (Braginskii, 1965). Similar expressions are applied to the charged grains, too. Here, λ is the Coulomb logarithm, Z the number of electronic charge, n_i the ion population density, k_B the Boltzmann constant,