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The Smaller Alignment Index (SALI) applied in a study of stellar orbits in barred galaxies potential models using the LP-VIcode

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ABSTRACT

The Smaller Alignment Index (SALI) is a mathematical tool, not yet conventional, for chaos detection in the phase space of Hamiltonian Dynamical Systems. The SALI values has temporal behaviors very specific to ordered or chaotic motions, what makes the distinction between order and chaos easily observable in these systems. In this paper, this method will be applied to the stability study of stellar orbits immersed in gravitational potential of barred galaxies, since the motion of a test particle in a rotating barred galaxy model is given by a Hamiltonian function. Extracting four parameter sets from the Manos and Athanassoula (2011) work and elaborating a different initial conditions set for each case, we were able to introduce another point of view of their stability study for two degrees of freedom. We have also introduced two new extreme models that corroborates with the conclusions that more axisymmetric bars create an environment with less chaos and that more massive bars create an environment with more chaos. Separate studies were carried out for prograde and retrograde orbits that showed that the retrograde orbits seem more conducive to chaos. To perform all the orbits integrations we used the LP-VIcode program.

1. Introduction

Galaxies can be classified according to the scheme proposed by Hubble (1926). This scheme separates the spiral galaxies into two types: regular spiral galaxies (S) and barred spiral galaxies (SB). Indeed, about 65% of disk galaxies show bar-like structures (Eskridge et al., 2000; Sheth et al., 2003). Between them, the characteristic of their bars vary considerably, from faint weak bars to prominent, strong and massive bars. In this work, we will focus on the study of gravitational potentials generated by a barred galaxy structure.

The classic way to study these barred galactic potentials is to observe the structure of the orbits that are supported by them. Stellar orbits supported by a galactic potential are the basic constituents of any galactic structure. Therefore, the study of the stellar orbits characteristics is of vital importance for the understanding of the formation and evolution of these structures. Also, the analysis of periodic orbits and their stability provides valuable information about galaxy structure. There is a bond between stable periodic orbits and regular motion, once they are surrounded by tori of quasi-periodic motion. Unstable periodic orbits, by their side, originates chaos (Manos and Athanassoula, 2011).

Bars are non-axisymmetric structures that can be mathematically modelled by the ellipsoidal density distribution proposed by Ferrers (1877). It has been extensively used in many works (e.g.:Manos and Athanassoula, 2011; Athanassoula et al., 1983; Pfenniger, 1984; Patsis, 2002; Patsis et al., 2003a; Skokos et al., 2002a; 2002b; Patsis et al., 2002; 2003b; Bountis et al., 2012).

Let us describe some studies related to galactic bars made in the last decades. Athanassoula et al. (1983) integrated orbits in a rigid potential composed of a prolate heterogeneous bar in an axysimmetric background. Varying the parameter set of the model, they studied the main families of periodic orbits and non-periodic orbits. It was shown that inside corotation, there are two main prograde families, one confined to small radii, and the second aligned with the bar, as well as a retrograde family. They also noted that more massive or more eccentric bars create more irregular motions, affecting the building of self-consistent bars. Outside corotation, analogs are found for the two main prograde families, as well as a $-1/1$ resonant family. In turn, Pfenniger (1984), focused on the dynamics in a 3D model. Here we highlight the importance of the appendix of this publication, where the author presented a very useful polynomial form for the Ferrers potential.

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Combes et al. (1990) presented a set of N-body simulations of collisionless disk galaxies, from which they concluded that all bars that develop in realistic models take, after some time, a more or less pronounced peanut-shape when seen edge-on. Olle and Pfenniger (1998) studied stability of the Lagrangian points and the vertical periodic orbits around them. Skokos et al. (2002a), Skokos et al. (2002b), Patsis et al. (2002) and Patsis et al. (2003b) published a series of articles studying periodic orbits in a potential model of barred galaxy seeking the influences of the system parameters on the nature of these orbits and their families. Kaufmann and Patsis (2005) suggested that for sufficiently large bar axial ratios, stable orbits having propeller shapes has great influence on the structure of the bar. In a paper published in 2011, Manos and Athanassoula (2011) computed the percentage between regular and chaotic orbits into four parameters sets of a potential model, where each set represented a different type of bar. Bountis et al. (2012) used probability density functions of sums of orbit coordinates, over time intervals of the order of one Hubble time, to distinguish weakly from strongly chaotic orbits in a barred galaxy model. Manos and Machado (2014) and Machado and Manos (2016) have written about the barred galaxies stability using time-dependent analytical potentials based on a N-body simulation of a strongly barred galaxy by extracting parameters of the simulation for certain times in the system evolution.

Zotos and Caranicolas (2016) using a model which consists of a harmonic oscillator and a spherical component, computed the percentages of chaotic orbits, as well as that of different types of regular orbits. They provide evidence that, besides the traditional x_1 orbital family, several other types of resonant orbits can support the bar structure. Also for sparse nuclei, fast rotating bars and high energy models, the structure is supported. On the contrary, weak bars, dense central nuclei, slowly rotating bars and low energy models favors the formation of nuclear rings.

Based on the discussion above, we noticed the importance of the study of both, regular and chaotic orbits to understand the structural stability of the galactic bars. Indeed, there are several methods to detect chaos in orbits. Some are based on frequencies (e.g.: Laskar, 1990; 1993; Carpintero and Aguilar, 1998; Wang et al., 2016), others are variational methods (e.g.: Skokos, 2001; Skokos et al., 2007; Skokos, 2010; Cincotta and Simó, 2000; Cincotta et al., 2003; Froeschlé et al., 1997; Lega and Froeschlé, 2002; Fouchard et al., 2002; Voglis et al., 1999; Contopoulos and Voglis, 1996; Sándor et al., 2004; Carpintero et al., 2014).

In this paper, we study the nature of the orbits immersed in analytical potentials with two degrees of freedom representing barred galaxies. In order to do this, we applied the Smaller Alignment Index (SALI), which is a variational method for distinguishing regular and chaotic motions in the phase space of Hamiltonian Dynamical Systems (Skokos, 2001). A rotating barred galaxy model is given by a Hamiltonian function, therefore it is a liable system to apply SALI.

The main purpose of this paper is to verify the bar influence on the stability of orbits immersed in gravitational analytical potentials, as well as to supply another point of view from the Manos and Athanassoula (2011) work, using the same galaxy models and setting up initial conditions in a different way. We also intent to study extreme situations by using two new models provided by us.

To perform the orbits integrations and the SALI calculation, we used a slight adaptation of the LP-VIcode program (Carpintero et al., 2014), which is a fully operational code, implemented in Fortran 77, that calculates efficiently 10 different chaos indicators for dynamic systems, regardless of the number of dimensions, where SALI is one of them.

2. Methodology

2.1. The Smaller Alignment Index (SALI)

The SALI method was introduced by Skokos (2001) and it is very efficient to distinguish between ordered and chaotic motion in Hamiltonian systems. In order to define SALI, let us consider a Hamiltonian flow of N degrees of freedom, an orbit in the $2N$ dimensional phase space with initial condition $x(0) = (x_1(0), \cdots, x_{2N}(0))$ and two deviation vector $\mathbf{w}_1(0)$,$\mathbf{w}_2(0)$ from the initial condition $x(0)$.

Consider that the deviation vectors are normalized each time step, because we will be only interested in this vectors directions. Mathematically, we use the notation $\widehat{\mathbf{w}}_i(t) = \frac{\mathbf{w}_i(t)}{\| \mathbf{w}_i(t) \|}$, where $i \in \{1, 2\}$ and $|| \cdot ||$ denotes the usual Euclidean norm.

Then, lets define the two following quantities:

(i) The Parallel Alignment Index:

$$d_{-}: = \| \widehat{\mathbf{w}}_1(t) - \widehat{\mathbf{w}}_2(t) \| \tag{1}$$

(ii) The Antiparallel Alignment Index:

$$d_{+}: = \| \widehat{\mathbf{w}}_1(t) + \widehat{\mathbf{w}}_2(t) \| \tag{2}$$

With this, we define The Smaller Alignment Index as:

$$SALI(t): = min\{d_{-}, d_{+}\} \tag{3}$$

Consequently it is evident that $SALI(t) \in [0, \sqrt{2}]$ and when $SALI = 0$ the two normalized vectors have the same direction, being equal or opposite.

It is possible to prove (Skokos, 2001; Skokos et al., 2002c; 2003; 2004) that in the case of chaotic orbits, the deviation vectors $\widehat{\mathbf{w}}_1(t)$ and $\widehat{\mathbf{w}}_2(t)$ align in the direction defined by the Maximum Lyapunov Exponent (MLE) and SALI (t) falls exponentially to zero:

$$SALI(t) \propto e^{-(L_1 - L_2)t} \tag{4}$$

with L_1 and L_2 the two largest Lyapunov Exponents.

Also, according to Skokos and Bountis (2012), when the behavior is ordered, the orbits develop on a phase space torus and eventually the vectors $\widehat{\mathbf{w}}_1(t)$ and $\widehat{\mathbf{w}}_2(t)$ fall in the torus tangent space, following a t^{-1} time dependence. In this case, the SALI oscillates at nonzero values (Skokos, 2001; Skokos et al., 2002c; 2003; 2004):

$$SALI(t) \approx constant > 0 \tag{5}$$

According to the above content, we have a clear distinction between ordered and chaotic behaviors using the SALI method in Hamiltonian systems.

2.2. Mathematical modeling of the gravitational potential

We can elect some analytical models for the bulge, the disk and the bar potentials, which together form the galactic potential model. Thus, the total potential can be written as:

$$\Phi_{Total} = \Phi_{Bulge} + \Phi_{Disk} + \Phi_{Bar} \tag{6}$$

Mirrored in Manos and Athanassoula (2011), we used the following analytical potentials for each component:

The bulge potential by the Plummer model (Plummer, 1911):

$$\Phi_{Bulge} = -\frac{GM_S}{\sqrt{x^2 + y^2 + z^2 + \epsilon^2}} \tag{7}$$

where ϵ is the scale-length of the bulge and M_S is its total mass, and G is the gravitational constant.

The disk potential by the Miyamoto–Nagai model (Miyamoto and Nagai, 1975):

$$\Phi_{Disk} = -\frac{GM_D}{\sqrt{x^2 + y^2 + (A + \sqrt{z^2 + B^2})^2}} \tag{8}$$

where M_D is the total disk mass, A and B are its horizontal and vertical