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Perturbation of mass accretion rate, associated acoustic geometry and stability analysis

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HIGHLIGHTS

- We investigate the stability of stationary integral solutions of an ideal irrotational fluid in a general static and spherically symmetric background, by studying the profile of the perturbation of the mass accretion rate.
- We consider low angular momentum axisymmetric accretion flows for three different accretion disk models and consider time dependent and radial linear perturbation of the mass accretion rate.
- First we show that the propagation of such perturbation can be determined by an effective 2×2 matrix, which has qualitatively similar acoustic causal properties as one obtains via the perturbation of the velocity potential.
- Next, using this matrix we analytically address the stability issues, for both standing and travelling wave configurations generated by the perturbation.
- Finally, based on this general formalism we briefly discuss the explicit example of the Schwarzschild spacetime and compare our results of stability with the existing literature, which instead address this problem via the perturbation of the velocity potential.

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ABSTRACT

We investigate the stability of stationary integral solutions of an ideal irrotational fluid in a general static and spherically symmetric background, by studying the profile of the perturbation of the mass accretion rate. We consider low angular momentum axisymmetric accretion flows for three different accretion disk models and consider time dependent and radial linear perturbation of the mass accretion rate. First we show that the propagation of such perturbation can be determined by an effective 2×2 matrix, which has qualitatively similar acoustic causal properties as one obtains via the perturbation of the velocity potential. Next, using this matrix we analytically address the stability issues, for both standing and travelling wave configurations generated by the perturbation. Finally, based on this general formalism we briefly discuss the explicit example of the Schwarzschild spacetime and compare our results of stability with the existing literature, which instead address this problem via the perturbation of the velocity potential.

1. Introduction

To obtain reliable spectral signatures of astrophysical black holes using a set of stationary transonic accretion solutions, it is necessary to ensure that such integral solutions are stable under perturbation (Kato et al., 1998), at least for an astrophysically relevant time scale. In Bollimpalli et al. (2015a), such stability was argued via demonstrating the natural emergence of the acoustic analogue geometry through perturbation of the mass accretion rate for radial Bondi flows with spherical symmetry (Michel, 1972; Bondi, 1952). Here we wish to extend this perturbation scheme to accommodate low angular momentum axisymmetric flows, assuming the fluid to be inviscid, irrotational and non self-gravitating, with three different disk models the axisymmetric accretion flow can have. Some preliminary results in this direction can be seen in Bollimpalli et al. (2015b).

On the analytical front, stationary flow solutions for the low angular momentum inviscid accretion has extensively been studied in the literature, see, e.g. Das and Czerny (2012); Fishbone and Moncrief (1976); Fukue (1983, 1987, 2004); Gaite (2006); Gammie and Popham (1998); Garlick (1979); Ho (1999); Igumenshchev and Abramowicz (1999); Illarionov and Sunyaev (1975);

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Illarionov (1988); Ipser and Lindblom (1992); Ipser (1994); Kafatos and Yang (1994); Kato et al. (1998); Liang and Thomson (1980) and references therein. Numerical works have also been reported for such flow configurations in Okuda et al. (2007); Okuda and Molteni (2012); Petterson et al. (1980); Pu et al. (2012); Sen and Ray (2014); Sponholz and Molteni (1994) and references therein.

In this context it should be emphasized that the concept of low angular momentum advective flow (where the inviscid assumption is justified) is not a theoretical abstraction. Sub-Keplerian flows are observed in nature. Such flow configurations may be observed for detached binary systems fed by accretion from OB stellar winds (Illarionov and Sunyaev, 1975; Liang and Nolan, 1984), semi-detached low-mass non-magnetic binaries (Bisikalo et al., 1998), and supermassive black holes fed by accretion from slowly rotating central stellar clusters (Illarionov, 1988; Ho, 1999) (see also the references therein). Even for a standard Keplerian accretion disc, turbulence may produce such low angular momentum flow, see, e.g. Igumenshchev and Abramowicz (1999) and references therein. Moreover, given a background spacetime, it is natural to expect a critical value of the specific angular momentum of the flow, below which there would not be any Keplerian orbits.

In addition to the analysis of the stationary transonic solutions, the stability properties of such flow has also been performed in various works, see, e.g. Bose et al. (2014); Chakrabarti (1989); Chaudhury et al. (2006); Das (2002); Das et al. (2003); Das and Czerny (2012); Fishbone and Moncrief (1976); Fukue (1983, 1987, 2004); Gaite (2006); Gammie and Popham (1998); Garlick (1979); Ho (1999); Igumenshchev and Abramowicz (1999); Illarionov and Sunyaev (1975); Illarionov (1988); Ipser and Lindblom (1992); Ipser (1994); Kafatos and Yang (1994); Kato et al. (1998); Liang and Thomson (1980); Liang and Nolan (1984); Lu (1985, 1986); Lu et al. (1995, 1997a, 1997b); Lu and Yuan (1998); Lu and Gu (2004); Mach and Malec (2008); Mach (2009); Michel (1972); Moncrief (1980) and references therein.

In the present work, we study the linear perturbation of the stationary transonic black hole accretion solutions through its connection to the emergence of the sonic geometry embedded within spacetime characterizing the background stationary flow (see Moncrief (1980); Unruh (1981); Barcelo et al. (2005), and references therein, for detailed discussion about analogue gravity phenomena).

The emergent analogue acoustic geometry in accretion astrophysics via perturbation of the potential of irrotational velocity flow and related stability issues was first studied extensively in Moncrief (1980), for spherical accretion. To the best of our knowledge, as of now the acoustic geometry associated with usual analogue gravity models has been obtained by perturbing the corresponding velocity potential of the background fluid flow. Instead, in the present work we discuss the sonic causal structure by perturbing the mass accretion rate associated with the infalling matter. The motivation behind this is obvious – the mass accretion rate is an astrophysically relevant and measurable quantity, and hence it is interesting to determine its profile perturbatively. Moreover, mass accretion rate is associated with both the density field and velocity field, which provide the full description of the flow. Therefore, perturbing the mass accretion rate leads to a wave equation that can shed light on stability of both the fields. In Bollimpalli et al. (2015a), such connection was demonstrated for spherical accretion, generalizing the non-relativistic results of Naskar et al. (2007). In this work, we wish to extend these earlier results for axisymmetric matter flow with nontrivial disk structures in general static and spherically symmetric spacetimes.

However, there is a crucial difference between the acoustic geometry we derive here with the same obtained via the perturbation of the velocity potential. We shall ignore any non-axisymmetric features of the perturbation corresponding to the mass accretion rate, and assume that the accretion rate is a function of the radial and the time coordinates only. Accordingly, the internal geometry through which the perturbation propagates has dimension two, instead of three or four. Nevertheless, we will see that the qualitative features regarding the causal structures of these two acoustic geometries remain the same, even though the mass accretion rate and the velocity potential are two very distinct quantities. To the best of our knowledge, such connection has not been reported in the existing literature.

For axisymmetric accretion, infalling matter can have three different geometric configurations – the conical, the constant height and the vertical equilibrium (at least for thin acrretion disks) models (see, e.g. Nag et al. (2012) and references therein). In subsequent sections we shall derive the general relativistic acoustic geometry through which the linear perturbation (corresponding to the mass accretion rate) propagates, for all these three models. We then briefly address the stability properties of the axisymmetric matter flows and reestablish the chief qualitative features reported earlier using different methods (Moncrief, 1980), for spherical accretion.

We shall use mostly positive signature for the metric and will set $c = 1 = G$ hereafter. We outlined the paper in the following manner. In Section 2 we briefly discuss the preliminaries and the equations describing the relativistic axisymmetric fluid flow in static and spherically symmetric spacetime for all the three disk models considered. In Section 3, we first discuss the stationary solutions of the flow and derive the critical point condition. Following this we perform the linear mass accretion rate perturbation and derive the effective 2×2 matrix through which the perturbation propagates. Using this in Section 4, we address the stability issues by deriving the profile of the perturbation of the mass accretion rate. In Section 5, we summarize our work with an outlook.

2. The basic constructions

We shall briefly mention here the basic ingredients and assumptions necessary for our calculations. Let us start with the metric for a general static and spherically symmetric spacetime

$$ds^2 = -g_{tt}(r)dt^2 + g_{rr}(r)dr^2 + r^2 d\Omega^2, \tag{1}$$

where $d\Omega^2$ is the line element of a unit 2-sphere.

Let us take an ideal and inviscid fluid with the energy-momentum tensor,

$$T_{\mu\nu} = (\epsilon + p)v_\mu v_\nu + p g_{\mu\nu}, \tag{2}$$

where ϵ and p are respectively the mass-energy density and pressure, v^μ is the fluid's four velocity normalized as $v^\mu v_\mu = -1$.

We shall consider axisymmetric radial flow on the equatorial plane, hence $v^\theta = 0$. The normalization condition thus provides

$$v^t = \sqrt{\frac{1 + g_{rr}v^2 + g_{\phi\phi}(v^\phi)^2}{g_{tt}}}, \tag{3}$$

where we have written $v^r \equiv v$. We assume that the fluid obeys the adiabatic equation of state $p = k\rho^\gamma$, where γ is adiabatic index and ρ is the fluid density. The specific enthalpy h of the fluid is given by $h = \frac{\epsilon + p}{\rho}$, so that

$$dh = Td\left(\frac{S}{\rho}\right) + \frac{dp}{\rho}, \tag{4}$$

where T and S are the temperature and entropy of the fluid respectively. Under isoentropic conditions, we can define the speed of the sound, c_s to be

$$c_s^2 = \left.\frac{\partial p}{\partial \epsilon}\right|_{dS=0}. \tag{5}$$