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Linearized stability of Reissner Nordstrom de-Sitter thin shell wormholes

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HIGHLIGHTS

- Matching two RN de-Sitter solutions across the singular surface.
- Linearized radial perturbations around static solutions.
- Stability analyses of thin shell wormholes are proposed.
- The regions of stability are greatly increased for large values of charge.
- The regions of stability are greatly increased for large values of Λ.

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ABSTRACT

Using the Darmois–Israel formalism the dynamical analysis of Reissner Nordstrom de-Sitter thin shell wormholes, at the wormhole throat, are determined by considering linearized radial perturbations around static solutions.

The region of stability in the presence of a large value of charge is significantly increased. Also, the region of stability in the presence of a positive large value of cosmological constant is significantly increased.

1. Introduction

The study of the dynamics of a shell separating two backgrounds in the context of general relativity has been developed in a powerful and direct formalism since the pioneer work of Israel (1966) and applied to the charged shell by Kuchar (1968). It has been applied to cosmology, mainly to inflation, Berezin et al. (1987), and to modeling the dynamics of the border between two regions in different states, like bubbles or between two given spaces, Sato (1988). The linearized stability analysis of spherical shells was carried out by several authors. For instance, Kim (1992) analyzed Schwarzschild-de Sitter wormholes, using the cut-and-paste construction.

The formalism was applied to bubbles, shells around stars and black holes, and in the construction of thin-shell wormholes (with spherical, plane and also cylindrical throats;see for example, Lobo and Crawford (2005), Visser (1996), Eiroa and Simeone (2004)). Besides, thin shells are associated to gravastars, for which the stability was also studied, Lobo and Arellano (2007). Poisson and Visser (1995) analyzed a thin-shell wormhole, constructed by pasting together two copies of the Schwarzschild solution.

This paper is organized as follows. In Section 2 the Darmois–Israel formalism is briefly reviewed. Match an interior RN de-Sitter spacetime to an exterior RN de-Sitter spacetime, the dynamical equations of thin shell wormholes are given in Section 3. The linearized stability analysis of wormholes is given in Section 4. A general conclusion is given in Section 5. Also adopt the units such that $c = G = 1$.

2. The Darmois–Israel Formalism

Consider two distinct spacetime manifolds M_+ and M_- with metrics given by $g^+_{\mu\nu}(x^\mu_+)$ and $g^-_{\mu\nu}(x^\mu_-)$, in terms of independently

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defined coordinate systems $x^{\mu}_{\pm}$. The manifolds are bounded by hypersurfaces Σ_{+} and Σ_{-}, respectively, with induced metrics $g^{\pm}_{ij}$. The hypersurfaces are isometric, i.e. $g^{+}_{ij}(\xi) = g^{-}_{ij}(\xi) = g_{ij}(\xi)$, in terms of the intrinsic coordinates, invariant under the isometry. A single manifold M is obtained by gluing together M_{+} and M_{-} at their boundaries, i.e. $M = M_{+} \cup M_{-}$, with the natural identification of the boundaries $\Sigma = \Sigma_{+} = \Sigma_{-}$. The second fundamental forms (extrinsic curvature) associated with the two sides of the shell are:

$$K^{\pm}_{ij} = -n^{\pm}_{\gamma}\left(\frac{\partial^2 x^{\gamma}}{\partial\xi^i\partial\xi^j} + \Gamma^{\gamma}_{\alpha\beta}\frac{\partial x^{\alpha}}{\partial\xi^i}\frac{\partial x^{\beta}}{\partial\xi^j}\right)\Big|_{\Sigma} \tag{1}$$

where $n^{\pm}_{\gamma}$ are the unit normal 4-vector to Σ in M, with $n_{\mu}n^{\mu} = 1$ and $n_{\mu}e^{\mu}_{i} = 0$. The Israel formalism requires that the normal point from M_{-} to M_{+}. For the case of a thin shell K_{ij} is not continuous across Σ, so that, the discontinuity in the second fundamental form is defined as $[K_{ij}] = K^{+}_{ij} - K^{-}_{ij}$. The Einstein equation determines the relations between the extrinsic curvature and the three dimensional intrinsic energy momentum tensor are given by the Lanczos equations,

$$S^{ij} = \frac{-1}{8\pi}([K_{ij}] - [K]g_{ij}) \tag{2}$$

where $[K]$ is the trace of $[K_{ij}]$ and S_{ij} is the surface stress-energy tensor on Σ. The first contracted Gauss- Kodazzi equation or the "Hamiltonian" constraint

$$G_{\mu\nu}n^{\mu}n^{\nu} = \frac{1}{2}(K^2 - K_{ij}K^{ij} - {}^3R), \tag{3}$$

with the Einstein equations provide the evolution identity

$$S^{ij}\bar{K}_{ij} = \left[-T_{\mu\nu}n^{\mu}n^{\nu} - \frac{\Lambda}{8\pi}\right]^{+}_{-}. \tag{4}$$

The convention, $[X] = X^{+} - X^{-}$, and $\bar{X} = \frac{1}{2}(X^{+} + X^{-})$, is used. The second contracted Gauss–Kodazzi equation or the "ADM" constraint,

$$G_{\mu\nu}e^{\mu}_{i}n^{\nu} = K^{j}_{i;j} - K_{,i}, \tag{5}$$

With the Lanczos equations gives the conservation identity

$$S^{i}_{j;i} = \left[T_{\mu\nu}e^{\mu}_{i}n^{\nu}\right]^{+}_{-}. \tag{6}$$

The surface stress-energy tensor may be written in terms of the surface energy density σ, and surface pressure p: $S^{i}_{j} = diag\cdot(-\sigma, p, p)$. For spherically symmetric thin shell, the Lanczos equations reduce to

$$\sigma = \frac{-1}{2\pi}[K^{\theta}_{\theta}] \tag{7}$$

$$p = \frac{1}{4\pi}([K^{\tau}_{\tau}] + [K^{\theta}_{\theta}]). \tag{8}$$

If the surface stress-energy terms are zero, the junction is denoted as a boundary surface. If surface stress terms are present, the junction is called a thin shell.

3. Dynamics of thin shell wormhole

The matching of two Reissner Nordstrom de-Sitter space–times of $M^{\pm}$, given by the following line elements:

$$ds^2_{\pm} = -F_{\pm}(r)dt^2 + F^{-1}_{\pm}(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{9}$$

with

$$F_{\pm} = 1 - \frac{2m_{\pm}}{r} + \frac{q^2_{\pm}}{r^2} - \frac{1}{3}\Lambda_{\pm}r^2$$

where $m_{\pm}$, $q_{\pm}$ and $\Lambda_{\pm}$ are the gravitational mass, the charge and the cosmological constant outside and inside the shell. The suffix '+' denotes a quantity evaluated just outside the shell and '−' just inside the shell. Let the equation of the shell be $r_{\pm} = R_{\pm}(\tau)$, the history of the shell is described by the hypersurface $x^{\alpha}_{\pm} = x^{\alpha}_{\pm}(\tau, \theta, \varphi)$, in the regions $M^{\pm}$, respectively; the function $R(\tau)$ describes the time evolution of the shell. The non-trivial components of the extrinsic curvature are given by

$$K^{\theta\pm}_{\theta} = K^{\varphi\pm}_{\varphi} = \frac{1}{R}\sqrt{F_{\pm} + \dot{R}^2} \tag{10}$$

$$K^{\tau\pm}_{\tau} = \frac{1}{\sqrt{F_{\pm} + \dot{R}^2}}\left(\frac{m_{\pm}}{R^2} - \frac{Q^2}{R^3} + \ddot{R} - \frac{1}{3}\Lambda R\right) \tag{11}$$

Note that $\dot{R} = dR/d\tau$, where the parameter τ measures proper time along the wormhole throat. Therefore, the Lanczos equations are given by

$$\sigma = \frac{-1}{2\pi R}\left[\sqrt{F_{\pm} + \dot{R}^2}\right] \tag{12}$$

$$p = \frac{1}{4\pi R}\left[\frac{1 - \frac{m_{\pm}}{R} - \frac{2}{3}\Lambda R^2 + \dot{R}^2 + R\ddot{R}}{\sqrt{F_{\pm} + \dot{R}^2}}\right] \tag{13}$$

Therefore, the energy conservation can be written in the form:

$$\dot{\sigma} = \frac{-2\dot{R}}{R}(\sigma + p) \tag{14}$$

In this equation, the first term corresponds to a change in the throat's internal energy, while the second term corresponds to the work done by the throat's internal forces. Rearranging equation (12) to get the equation of motion of thin shell wormhole,

$$\sqrt{F_{-} + \dot{R}^2} - \sqrt{F_{+} + \dot{R}^2} = \frac{M}{R} \tag{15}$$

where $M = \sigma A$ is the rest mass of the shell, ($A = 4\pi R^2$). This equation can be written in the form

$$\dot{R}^2 + V(R) = 0 \tag{16}$$

where

$$V(R) = \frac{-M^2}{4R^2} + \frac{1}{2}(F_{-} + F_{+}) - \frac{R^2}{4M^2}(F_{-} - F_{+})^2 \tag{17}$$

is the effective potential. This dynamical equation completely determines the motion of the wormhole throat.

4. Stability analyses

From (14), with $p = p(\sigma)$, the conservation equation is

$$\int\frac{dR}{R} = -\frac{1}{2}\int\frac{d\sigma}{\sigma + p(\sigma)} \tag{18}$$

This relationship may then be formally inverted to: $p = p(R)$. Then, the dynamical equation (15) can be written in the form

$$\dot{R}^2 + F_{\pm} - (2\pi R\sigma)^2 = 0 \tag{19}$$

where

$$V(R) = F_{\pm} - (2\pi R\sigma)^2 \tag{20}$$

In the case of the static solution where, $\dot{R} = \ddot{R} = 0$, and the characteristic constants are $\sigma_{\circ}$, $R_{\circ}$, and $p_{\circ}$, the Eqs. (12) and (13) are:

$$\sigma_{\circ} = \frac{-1}{2\pi R_{\circ}}\sqrt{F_{\circ\pm}} \tag{21}$$