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Dimensions and equilibrium structures of the primary component of the nonsynchronous binary systems

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HIGHLIGHTS

- We study dimensions and structures of primary components of nonsynchronous binaries.
- First approximation theory of Limber (1963) has been used for this study.
- Nonsynchronism affects dimension and structures of primary components of binaries.
- Effects increases as difference of rotational and orbital velocities increase.
- Effect of rotation is more than the effect due to revolution and tidal forces.

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ABSTRACT

Rotating stars and stars in the synchronous binaries have been extensively studied in literature. However, there are only few studies that have investigated the problems of the nonsynchronous binaries. In the present paper, we have made an attempt to study the various dimensions and equilibrium structures of the primary component of the nonsynchronous binaries. We have used the first approximation theory of Limber (1963) along with the methodology as that proposed by Mohan and Saxena (1983) for the present study. The objective of this paper is to check the effect of nonsynchronism on the various dimensions and equilibrium structures of the primary components of the binary systems. The results of the present study shows that there is change in the dimensions and equilibrium structures of the primary component of the binary systems due to nonsynchronism, and this change is more appreciable when the difference between the angular velocities of rotation and revolution is large.

1. Introduction

In the major cases of astrophysical significance only synchronous rotation is assumed. This is supported by the theoretical calculations of the Zahn (1977) that considers the tidal interaction between the components of the binary system to be responsible for synchronization. There are various studies in the literature that have investigated – in detail – the various problems of the synchronous binaries. Authors such as Eggleton (1983), Jackson (1970), Kopal (1959), Kuiper and Johnson (1956), Lal et al. (2006, 2009), Martin (1970), Mochnacki (1984), Mohan and Saxena (1983, 1990), Plavec and Kratochvil (1964) and SirotKin and Kim (2009) have made a significant progress in investigating the various problems of the dimensions and/or equilibrium structures of the synchronous binary systems.

Since the discovery of the nonsynchronous rotation (when the angular velocity of one or both components of the binary system differs from the orbital angular velocity) by Schlesinger in the spectroscopic binary δ Lib in 1909 and in the eclipsing binary λ Tau in 1910 (c.f. Tassoul 1978), the nonsynchronous rotation has been detected in numerous binary systems of various types. The nonsynchronism in binary systems has been the subject of several investigations. The literature has been reviewed by Struve (1950), Levato (1974), Habets and Zwaan (1989), Meibom et al. (2006) and van Hamme and Wilson (1990). The effect of the nonsynchronism on the dimensions and shapes of the binary systems was first studied by Plavec (1958). Further investigations were then carried by Csataryova and Skopal (2005), Kruszewski (1963), Limber (1963), Lubow (1979), Naylor (1972) and Sepinsky et al. (2007).

The effect of the nonsynchronism has been neglected as it not only complicates the analysis but the theory of synchronism has been successful for long in explaining the various observational aspects of the binaries. Also, the nonsynchronism in binary systems

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induces time – dependent oscillations in the atmospheres of the component stars. As a result, these components can no longer be treated as static with respect to a rotating frame of reference, unless the time scale of the oscillatory motions is sufficiently long as that compared to the dynamical time scale of the star. The validity of this approximation was first discussed by Limber (1963) for the case of nonsynchronous binary stars. He concluded that as long as the rotational angular velocities of the stars do not deviate considerably from synchronicity, the components may be approximated as static and their shapes may be determined by the instantaneous equipotential surfaces of the binary. Limber (1963) also found that the approximation is valid in the limiting case of stars rotating with angular velocities close to the break up angular velocity due to the dominance of the centrifugal force over the gravitational forces.

Mohan and Saxena (1983) have studied the equilibrium structures of the polytropic models of the uniformly rotating stars and stars in the binary systems. In this study authors have used the equation of the Roche equipotential as that derived by Kopal (1972) for synchronous binaries. They have used this equation to study the equilibrium structures of the uniformly rotating single stars, purely tidally distorted stars (no rotation) and synchronous binaries. Furthermore, authors have used the same equation for studying the nonsynchronous binaries (they have assumed that for nonsynchronous binaries, $2n \neq q+1$ where n and q are the parameters of distortion due to rotation and tidal forces, respectively). However, it is not justified in the framework of the assumptions considered in their study because the basic equipotential equation that the authors have used, has been derived for synchronous binaries (Kopal 1972), and not for nonsynchronous binaries. This methodology to study the equilibrium structures of nonsynchronous binaries has been subsequently used by Mohan et al. (1990) and Lal et al. (2006). However, using the first approximation theory of Limber (1963) that considers the more general case of nonsynchronous binaries (single rotating stars and synchronous binaries are the particular cases of this theory), we can justify the use of the Roche equipotential for the nonsynchronous binaries as that done by Mohan and Saxena (1983).

Keeping these factors in view, in the present paper, we have studied in detail the various dimensions and equilibrium structures of the primary component of the nonsynchronous binaries. For this purpose, we have used the first approximation theory of Limber (1963) that represents the more general case of the nonsynchronous binaries. We have also used the methodology as that proposed by Mohan and Saxena (1983) that utilizes the averaging technique of Kippenhahn and Thomas (1970) along with the certain results on the Roche equipotential as that given by Kopal (1972). The objective of the present paper is to check that how the nonsynchronism affects the various dimensions and equilibrium structures of the primary component of the nonsynchronous binaries.

The paper is organized as follows: The expression for the Roche equipotential of the nonsynchronous binary systems has been obtained in Section 2. This expression of the Roche equipotential has been then used in Section 3 to obtain the expressions for the various dimensions of the primary component of the nonsynchronous binaries. In Section 4, the equations governing the equilibrium structures of the polytropic models of the primary component of the nonsynchronous binary systems have been obtained. The expressions for the volumes and surface areas of such polytropic models of the stars are obtained in Section 5. Numerical computations have been performed in Section 6 to compute the various dimensions and equilibrium structures of the primary component of the nonsynchronous binaries. Certain conclusions of astrophysical importance have been drawn in the final Section 7.

Table 1
Various parameters used in the manuscript.

S. no	Parameter	Definition
1	n	The rotation parameter that represents distortions due to revolution
2	n_1	The rotation parameter that represents distortions due to rotation
3	q	The tidal parameter that represents distortions due to tidal effects
4	x_1	First Lagrangian point (Point radius) of the primary component
5	ξ_1	The value of critical Roche equipotential at first Lagrangian point (Roche limit)
6	x'_1	Back radius of the primary component
7	y_2	Side radius of the primary component
8	x_2	x – coordinate corresponding to the side radius of the primary component
9	z_1	Pole radius of the primary component
10	ϕ	The angle at which the resulting RES intersects the $X-$ axis at first Lagrangian point
11	V	Volume of the primary component
12	S	Surface area of the primary component

2. Roche equipotential of the nonsynchronous binaries

Consider a binary system in which the two components are rotating about their axis of rotation and also revolving about an axis passing through the center of mass of the system. Let M_1 and M_2 be the masses of the two components (primary and secondary, respectively) of the binary system that are separated by distance D. Suppose that the position of the two components of such a binary system is referred to a frame of reference with origin at the center of gravity of the primary star, $X-$ axis along the line joining the mass centers of the two stars, $Z-$ axis perpendicular to the plane of the orbit of the two components, and which rotates with the constant angular velocity Ω.

Let $r_1 = \sqrt{x^2+y^2+z^2}$, $r_2 = \sqrt{(D-x)^2+y^2+z^2}$ and $r = \sqrt{(x-d_1)^2+y^2+z^2}$ represent the distances of a point P(x, y, z) from the centers of gravity of the primary star, secondary star and the center of gravity C $((d_1, 0, 0)$ where $d_1 = M_2D/(M_1+M_2))$ of the system, respectively. Let ω denote the angular velocity of revolution of the system about a line parallel to $Z-$ axis which passes through the center of gravity C of the system and is perpendicular to the $XY-$ plane. Following the first approximation theory of Limber (1963), the two components of the binary system are assumed to be Roche models (This approximation is reasonably valid for the stars that have high degree of central condensation. According to Limber (1963), in any case this assumption can be replaced by more realistic models of stars without any fundamental changes in the theory). Also, it is assumed that the actual mass motions within the primary component of the binary system can be included for the most part in a simple uniform rotation about its center (The assumption of uniform rotation is also not fully justified. The work of Spruit and Phinney (1998) and Spruit (1999) give evidence in support of the uniform rotation; whereas, Popper and Plavec (1976) and Domiciano de Souza et al. (2003) have claimed differential rotation for single and binary stars. However, to avoid complexity in obtaining the final expression for the Roche equipotential surface, the star in a question is assumed to be rotating uniformly).

Following Limber (1963) and Kopal (1972), for such a system the total potential at an arbitrary point P, in nondimensional form can be written as

$$\psi^* = \frac{1}{r_1^*} + q\left(\frac{1}{r_2^*} - \frac{\omega^{*2}}{1+q}x^*\right) + \frac{\Omega^{*2}}{2}(x^{*2}+y^{*2}) + \frac{\omega^{*2}}{2}\left(\frac{q}{1+q}\right)^2 \quad (2.1)$$