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Constraints on Dark Energy state equation with varying pivoting redshift

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HIGHLIGHTS

- We assume the DE state equations $w(a) = w_0 + w_a(a_p - a)$.
- We study the dependence of w_0 and w_a on $a_p = 1/(1 + z_p)$ by using COSMOMC code.
- We consider both massless and massive neutrinos.
- The z_p values at which w_0 and w_a become independent differs in the two cases.
- At these z_p, constraints on w_0 are narrower than at other redshifts.

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ABSTRACT

We assume the DE state equations $w(a) = w_0 + w_a(a_p - a)$, and study the dependence of the constraints on w_0 and w_a coefficients on the pivoting redshift $1 + z_p = 1/a_p$. Coefficients are fitted to data including WMAP7, SNIa (Union 2.1), BAO's (including WiggleZ and SDSS results) and H_0 constraints. The fitting algorithm is CosmoMC. We find specific differences between the cases when ν-mass is allowed or disregarded. More in detail: (i) The z_p value yielding uncorrelated constraints on w_0 and w_a is different in the two cases, holding ~ 0.25 and ~ 0.35, respectively. (ii) If we consider the intervals allowed to w_0, we find that they shift when z_p increases, in opposite directions for vanishing or allowed ν-mass. This leads to no overlap between 1σ intervals already at $z_p > \sim 0.4$. (iii) The known effect that a more negative state parameter is required to allow for ν mass displays its effects on w_a, rather than on w_0. (iv) The w_0–w_a constraints found by using any pivot z_p can be translated into constraints holding at a specific z_p value (0 or the z_p where errors are uncorrelated). When we do so, error ellipses exhibit a satisfactory overlap.

1. Introduction

Owing to the conceptual problems of ΛCDM, a number of options for Dark Energy (DE) nature have been considered. In particular, DE could be a scalar field, necessarily self–interacting and possibly interacting with Dark Matter Damour et al., 1990; Wetterich, 1995; Amendola, 2000; Amendola and Quercellini, 2001; Dalal et al., 2001; Amendola and Tocchi Valentini, 2002; Mainini and Bonometto, 2004; Maccio' et al., 2004; Mainini and Bonometto, 2006, 2007; Bento and Bertolami, 2009; Bento et al., 2008; Zimdahl et al., 2001; del Campo et al., 2006; Wei and Zhang, 2007; Amendola et al., 2007a,b,c; Guo et al., 2007; Caldera-Cabral et al., 2009; Pettorino et al., 2012, or just a phenomenological consequence of large scale GR violations (Capozziello et al., 2006; Amendola et al., 2007a,b,c; Creminelli et al., 2009; Park et al., 2010; Bloomfield and Flanagan, 2012). But neither these options, nor still more exhotic hypotheses (Tomita, 2000; Celerier, 2000; Tomita, 2001; Iguchi et al., 2002; Jimenez et al., 2012), led to appreciable improvements of the fit between theory and data (Colombo et al., 2009; Mainini, 2009; Kristiansen et al., 2010).

The problem has then been tackled from the phenomenological side, by testing whether any linear $w(a)$, different from $w(a) \equiv -1$, improves data fits. A possible option amounts then to express the linear laws through the equations

$$w(a) = w_0 + w_a(1 - a) \qquad (1)$$

aiming then at testing how various sets of data yield constraints on w_0 and w_a. Here a is the scale factor, normalized to unity at the

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present time. In the literature, this expression for $w(a)$ was first used by Chevallier and Polarski (2001).

The same linear laws can be expressed also through the equations

$$w(a) = w_{0,a_p} + w_{a,a_p}(a_p - a), \tag{2}$$

which differ from (1) for selecting a non-vanishing pivoting redshift

$$z_p = 1/a_p - 1, \tag{3}$$

while we put an extra index to the linear coefficients $w_{0,a_p},\ w_{a,a_p}$ to put in evidence that, when changing z_p, their values change. The straight lines defined by Eq. (1) and Eq. (2) are however the same: any Eq. (2) turns into an Eq. (1) if we set

$$w_{0,a_p} = w_0 - w_a(a_p - 1) \tag{4}$$

and $w_{a,a_p} = w_a$. Notice that this last identity does not imply that limits on w_{a,a_p} are independent from the pivoting redshift. In the sequel, whenever this causes no confusion, we shall however follow the common use and call $w_0,\ w_a$ the two parameters in any expression (2).

Linear laws can be fitted to data by using different a_p values. Here we aim at testing, first of all, how compatible are results obtained when varying the pivoting redshift.

We shall do so in two cases: either neglecting or allowing the option that $M_\nu = \sum_\nu m_\nu \neq 0$ (the sum is extended to the mass eigenvalues for 3 standard neutrino flavors). Let us also remind that the neutrino density parameter

$$\Omega_\nu h^2 = 1.08 \times 10^{-2}(M_\nu/\mathrm{eV})(T_{0\gamma}/2.73\ \mathrm{K})^3, \tag{5}$$

so that, when the dark matter reduced density parameter $\omega_c = \Omega_c h^2$ is assigned, the neutrino fraction $f_\nu = \Omega_\nu/\Omega_c$ immediately follows.

The value of a_p can be selected so to have uncorrelated phenomenological constraints on w_0 and w_a. Here we also wish to put in evidence that: (i) the pivoting redshift yielding uncorrelated constraints is different, if $f_\nu \equiv 0$ or can be $\neq 0$; (ii) also the dependence on a_p of the w_0 interval compatible with data depends on the above option.

We expect that the DE state parameter w takes lower values, even in the phantom domain, when M_ν is allowed. We shall test how this occurs, when we consider a wide set of data (see below). In particular, by allowing for (linearly) variable w, we can test whether data require a constantly low w_0 or a progressively decreasing law, set by a negative w_a.

In the recent literature, the set of linear $w(a)$ has also been parametrized by using the values taken by w at $z = 0$ and at a higher redshift, e.g. $z = 0.5$. In spite of advantages of this parametrization (Wang, 2008), quite a few authors still keep to the old one. We plan to deepen the relation with such approach in further work.

2. Results for $z_p = 0$.

Let us then report, first of all, the results of Monte Carlo fits of DE state equations *vs.* data, performed by using the algorithm CosmoMC[1] (Lewis et al., 2000; Lewis and Bridle, 2012, May 2010 version); the CosmoMC code was integrated with the first version of the PPF module[2] for CAMB[3] (Lewis et al., 2000; Fang et al., 2008). Fits were performed in respect to the following parameters: $w_0,\ w_a$ (in Eq. 2) and $\omega_b = \Omega_b h^2,\ \omega_c = \Omega_c h^2,\ \theta = 100 l_s/l_d,\ \tau,\ n_s,\ \log A,\ A_{SZ}$, plus f_ν when needed (respectively: reduced baryon density parameter, reduced CDM density parameter, 100 times the ratio between sound horizon at recombination and its angular diameter distance, optical depth due to reionization, primeval spectral index, logarithmic fluctuation amplitude with pivoting scale 0.05 Mpc^{-1}, SZ template normalization, neutrino fraction as defined below; h is the Hubble parameter in units of 100 (km/s)/Mpc). We however kept $\Omega_k = 0$.

Our data set includes CMB data from WMAP7,[4] supernovae from Union2.1 survey (Suzuki, 2012, option with no systematic errors), WiggleZ and SDSS BAO's data (Blake et al., 2011; Percival, 2010), HST data (Riess et al., 2009) and CMB lensing as provided by CosmoMC. We use different combinations of these data, as suitably detailed below.

In Fig. 1 we show 1σ and 2σ contours for the marginalized likelihood on the w_0–w_a plane, when $z_p = 0$, for the sets of data indicated in the frame. In comparison with the analogous curves shown in WMAP7 report Komatsu et al., 2011 for $f_\nu = 0$, our ellipses are slightly displaced towards more negative w_0 and greater w_a. The ranges found are closer to the Union 2.1 report by Suzuki (2012).

To gauge the widening of w_0 and w_a intervals when $M_\nu \neq 0$ is allowed, we kept the same abscissa and ordinate ranges in both sides. The widening is confirmed when fully marginalizing in respect to all other parameters, as is shown in Table 1. Let us however notice that, when $M_\nu \equiv 0$ is required, the inclusion of BAO and/or HST data in top of CMB, causes a displacement towards smaller values of the mean w_0 and an increase of w_a. These shifts – just below 1σ – do not occur (or are much smaller) when $M_\nu \neq 0$.

In Fig. 2 we show the likelihood distributions on the f_ν–w_0 and f_ν–w_a planes, outlining a progressive delving of w_a into the negative domain when f_ν shifts from 0 to 0.04 (i.e., when M_ν shifts from 0 to $\sim$0.60 eV). The known result that a greater M_ν is allowed, when w delves in the phantom area, therefore affects w_a rather than w_0, so indicating that, to soften M_ν limits, it seems preferable that w shifts below -1 just when $z > 0$.

More in detail, if constant w models only are considered, as in Komatsu et al. (2011) or De Bernardis et al. (2008), one finds that, to compensate a massive neutrino component, DE density fading more rapidly than in ΛCDM, as z increases, is favored. As is known, even for $M_\nu \sim 0.1$ eV, neutrino derelativization is complete before $z = 100$. Since then, the whole linear fluctuation spectrum evolves $\propto (1+z)^{-1}$ until the spectral growth is slowed down by DE acquiring a significant density. If w is constant and < -1, DE density becomes significant later than in ΛCDM; a later slow down compensates the spectral depression, which is one of the consequences of neutrino mass. For instance, De Bernardis et al., 2008 found a constant $w = -1.12 \pm 0.09$. With our wider dataset, we found $w = -1.11^{+0.05}_{-0.04}$, an almost coincident result, apart of a halvened (1σ) errorbar.

When w linear variations are allowed, errors become greater, as expected. The central point of w_0 ($\simeq -1.07$) however rises up to the 1σ limit for constant–w models, while w tends to become negative because of w_a. This means a different timing in the reduced slowing down. A natural guess is that data coming from the epoch when DE starts to become significant, e.g. WiggleZ data, are better fitted by an early spectrum not only higher than ΛCDM, but even higher than a $w \simeq -1.12$ phantom model.

3. Results for $z_p \neq 0$

Let us then consider the fits when pivoting redshifts $z_p \neq 0$ are considered. In Fig. 3 we overlap the 2σ contour ellipses on the w_0–w_a plane, for $z_p = 0$, 0.25, 0.35 and 0.5, both for $f_\nu = 0$ (l.h.s.) and $\neq 0$ (r.h.s.). For the sake of clarity, we consider only the full set of observational constraints (CMB+SN+BAO+HST). The ellipses exhibit a progressive straightening of the symmetry axes and w_0–w_a errors

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[1] http://www.cosmologist.info/cosmomc

[2] camb.info/ppf

[3] http://www.camb.info/

[4] Provided by the website lambda.gsfc.nasa.gov