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Gravitational wave energy emission and detection rates of Primordial Black Hole hyperbolic encounters

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ABSTRACT

We describe in detail gravitational wave bursts from Primordial Black Hole (PBH) hyperbolic encounters. The bursts are one-time events, with the bulk of the released energy happening during the closest approach, which can be emitted in frequencies that could be within the range of both LIGO (10–1000 Hz) and LISA (10^{-6}–1 Hz). Furthermore, we correct the results for the power spectrum of hyperbolic encounters found in the literature and present new exact and approximate expressions for the peak frequency of the emission. Note that these GW bursts from hyperbolic encounters between PBH are complementary to the GW emission from the bounded orbits of BHB mergers detected by LIGO, and help breaking degeneracies in the determination of the PBH mass, spin and spatial distributions.

1. Introduction

In the last couple of years, Advanced LIGO has brought forward a new era of Gravitational Wave Astronomy, with the observation of several massive Black Hole (BH) merger events [1–5], as well as the recent detection of a binary neutron star inspiral [6], which opened the era of Multimessenger Astronomy [7]. In the case of the BH mergers, the signal corresponds to the inspiralling of two massive BHs in approximately circular orbits, and the emission of gravitational waves (GW) leading to the final merger is in excellent agreement with the expected result from General Relativity (GR).

Since the large number of massive BH binaries were unexpected, see however [8], they seem to hint at a new population of massive BHs. This opens fertile ground for speculating that AdvLIGO could have observed Primordial Black Holes (PBH) as a significant fraction of Cold Dark Matter (CDM) [9–11], thus offering a viable alternative to modifications of gravity or exotic particles beyond the standard model. For the effect of primordial black holes on structure formation see also Refs. [12–14]. Moreover, the PBH might also serve as the seeds for the Supermassive Black Holes (SMBH) located in the centers of the galaxies [15,16], as well as providing coherent explanations for a host of other problems in the standard cosmological model, such as the too-big-to-fail and the missing-satellite problems [17].

In the case of micro-clustered PBH, as proposed in Ref. [16], one would expect that a large fraction of BH encounters will not end up generating bounded systems, but instead just a hyperbolic encounter with emission of GW bremsstrahlung. This is what actually happens if the relative distance or velocity of the two bodies is too high for a BH capture. This case has already been studied in the past in Refs. [18], and [19,20], for parabolic and hyperbolic encounters respectively. These events produce enormous *bursts* of gravitational waves, which can in principle be bright enough to be detected at cosmological distances [21,22], see also [23] for detection rate estimates from parabolic encounters. The specific waveform of the GW event can be obtained analytically, without the need for computationally-expensive numerical relativity codes, and used to match the coincident event in the three LIGO+Virgo detectors.

It should be noted, however, that the full expression for the power emitted in GW, i.e. the Fourier spectrum of the GW emission, in the case of a hyperbolic encounter given by Eq. (3.11) of Ref. [20] is incorrect.[1] This minor error seems to have escaped the authors as their expression gives the correct limit in the case of parabolic orbits, and in their paper they only considered eccentricities higher than two, due to the choice of the parameters for the astrophysical system considered. We will present the correct expressions, which also have all the correct limits and no singularities, in the Appendix.

Furthermore, in the micro-clustered PBH scenario [17], we note that for hyperbolic encounters, the characteristic time parameters, as well as the waveform, of the GW emission are very different from those of inspiralling PBH binaries. Furthermore, they provide complementary information which can in principle be used to

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[1] This can easily be checked, since for eccentricity $e = 2$ there is an unphysical pole in the emitted power.

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break degeneracies and estimate the mass, spin and spatial distribution of PBHs as a function of redshift.

The main characteristic of hyperbolic encounters, and the main reason they could be so useful, is that they are one-time events where the main bulk of the energy is released near the periastron. These events also have a uniquely identifiable peak frequency which depends on just the total mass of the system M, the relative velocity v_0 and the impact parameter b. It should be noted that inspiralling and merging PBH has already been studied in the literature, see for example Refs. [10,24], where the estimated rate was in the range of tens of events/year/Gpc^3 for $M_{\rm PBH} \sim \mathcal{O}(10-100)\,M_\odot$. In Ref. [25] is was shown that, within the range of parameters of the micro-clustered PBH scenario [10,16], the rate of GW burst events in the millisecond range is of the same order of magnitude, if somewhat lower. Nevertheless, for a large fraction of hyperbolic events, the maximum amplitude is well within the noise of the LIGO detectors and without a proper waveform analysis will be very difficult to detect, while in BH spiraling events due to its periodic nature an long duration are much easier to detect.

As described in Ref. [25], hyperbolic encounter events would be detected by future gravitational wave experiments as bursts with a characteristic frequency at peak strain amplitude. Actually, AdvLIGO has already reported a few events of this type, which were then attributed to noise in the detectors [26]. However, events from hyperbolic encounters of PBH create shapes similar to the "tear drop glitch" analyzed in Ref. [27].

Therefore, in this paper we continue our analysis of those events, under the assumption that they are actually PBH hyperbolic encounters as their time–frequency profiles could shed light in the understanding of the AdvLIGO glitches. Finally, if it turns out that the glitches currently observed in AdvLIGO indeed originate from PBH hyperbolic encounters, then this fact could be used to determine the parameters describing the PBHs themselves, i.e. their spatial distribution, velocity and mass.

The layout of our manuscript is as follows: In Section 2 we discuss and review the basic relations that determine the geometry and physics of hyperbolic encounters, while in Section 3 we present the corrected expressions for the power spectrum for the emission but also the new analytic expressions for the frequency at peak amplitude. In Section 4 we present the observables that could be extracted from this system while we summarize and present our conclusions in Section 5.

2. Basic relations and constraints

Consider a hyperbolic encounter between a massive body m_2 with asymptotic velocity v_0 against a compact mass m_1. The total mass is given by $M = m_1 + m_2$ and the reduced mass is $\mu = m_1 m_2/M$. Let us assume an impact parameter b as in Fig. 1. Then, the eccentricity of the hyperbolic orbit is given by

$$e \equiv \sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{b^2 v_0^4}{G^2M^2}} > 1\,. \tag{1}$$

The orbital trajectory is characterized in polar coordinates by

$$r(\varphi) = \frac{b\,\sin\varphi_0}{\cos(\varphi-\varphi_0)-\cos\varphi_0} = \frac{a\,(e^2-1)}{1+e\cos(\varphi-\varphi_0)}\,, \tag{2}$$

where the relation between the impact parameter b and the semi-major axis a is given by (1), and the angle φ_0 is given by

$$\varphi_0 = \arccos\left(-\frac{1}{e}\right)\,, \tag{3}$$

while the distance of maximum proximity is given by

$$r_{\rm min} = a\,(e-1) = b\sqrt{\frac{e-1}{e+1}} > R_s \equiv \frac{2GM}{c^2}\,. \tag{4}$$

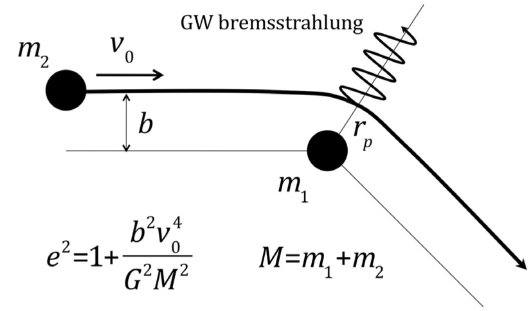

Fig. 1. The scattering of one BH of mass m_2 on another of mass m_1 induces the emission of gravitational waves which is maximal at the point of closest approach, r_p.

Conservation of angular momentum implies $b\,v_0 = r_{\rm min} v_{\rm max}$. We must impose that $v_{\rm max} < c$ or

$$\beta \equiv \frac{v_0}{c} < \sqrt{\frac{e-1}{e+1}}\,, \tag{5}$$

which substituted into (1) gives

$$b > R_s\,\frac{(e+1)^{3/2}}{2(e-1)^{1/2}}\,, \tag{6}$$

which is a factor $(e+1)/2$ stronger than (4).

2.1. Amplitude and power emitted in GW

The reduced quadrupole moment of the system is given by

$$Q_{ij} = \mu\, r^2(\varphi)\begin{pmatrix} 3\cos^2\varphi-1 & 3\cos\varphi\sin\varphi & 0\\ 3\cos\varphi\sin\varphi & 3\sin^2\varphi-1 & 0\\ 0 & 0 & -1\end{pmatrix}\,. \tag{7}$$

and the power emitted in GW is then given by

$$P = \frac{dE}{dt} = -\frac{G}{45c^5}\langle \dddot{Q}_{ij}\dddot{Q}^{ij}\rangle = \frac{32G\mu^2 v_0^6}{45c^5\,b^2}\,f(\varphi,\,e)\,, \tag{8}$$

$$f(\varphi,\,e) = \frac{3(1+e\,\cos(\varphi-\varphi_0))^4}{8(e^2-1)^4}\Big[24+13\,e^2 + 48\,e\,\cos(\varphi-\varphi_0) + 11\,e^2\cos 2(\varphi-\varphi_0)\Big] \tag{9}$$

and the strain amplitude by

$$h_c = \frac{2G}{R\,c^4}\langle \ddot{Q}_{ij}\ddot{Q}^{ij}\rangle^{1/2}_{i,j=1,2} = \frac{2G\mu\,v_0^2}{R\,c^4}\,g(\varphi,\,e)\,, \tag{10}$$

$$g(\varphi,\,e) = \frac{\sqrt{2}}{e^2-1}\Big[36+59\,e^2+10\,e^4 + (108+47\,e^2)\,e\,\cos(\varphi-\varphi_0) + 59\,e^2\cos 2(\varphi-\varphi_0) + 9\,e^3\cos 3(\varphi-\varphi_0)\Big]^{1/2} \tag{11}$$

where $f(\varphi,\,e)$ and $g(\varphi,\,e)$ are complicated bell-shaped functions of the angle φ, symmetric around φ_0, see Fig. 2. The maximum values occur for $\varphi = \varphi_0$, and only depend on the eccentricity of the orbit,

$$f_{\rm max}(e) = \frac{9(e+1)^2}{(e-1)^4}\,, \tag{12}$$

$$g_{\rm max}(e) = \frac{2}{e-1}\sqrt{18(e+1)+5e^2}\,. \tag{13}$$