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Implications of minimum and maximum length scales in cosmology

Seyen Kouwn

Department of Physics, BK21 Physics Research Division, Institute of Basic Science, Sungkyunkwan University, Suwon 440-746, Republic of Korea

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ABSTRACT

We investigate the cosmological implications of the generalized and extended uncertainty principle (GEUP), and whether it could provide an explanation for the dark energy. The consequence of the GEUP is the existence of a minimum and a maximum length, which can in turn modify the entropy area law and also modify the Friedmann equation. The cosmological consequences are studied by paying particular attention to the role of these lengths. We find that the theory allows a cosmological evolution where the radiation- and matter-dominated epochs are followed by a long period of virtually constant dark energy, that closely mimics the ΛCDM model. The main cause of the current acceleration arises from the maximum length scale β, governed by the relation $\Lambda \sim -\beta^{-1}W(-\beta^{-1})$. Using recent observational data (the Hubble parameters, type Ia supernovae, and baryon acoustic oscillations, together with the Planck or WMAP 9-year data of the cosmic microwave background radiation), we estimate constraints to the minimum length scale $\alpha \lesssim 10^{81}$ and the maximum length scale $\beta \sim -10^{-2}$.

1. Introduction

The observation that the universe is accelerating [1,2] has generated extensive investigations aiming to establish its theoretical foundation. A promising possible explanation involves invoking the cosmological constant Λ, which is related to the vacuum energy density. For consistency with existing observations, Λ must be very small, on the scale of $\sim 10^{-120}$, orders of magnitude smaller than the Planck scale M_p. However, the exact value gives rise to the "cosmological constant problem" [3]. Another possibility is a dynamic dark energy model [4–6], in which the cosmological constant varies dynamically. Arguably, observational data favor dynamical dark energy models over the standard ΛCDM model [7,8]. One possible way to describe dynamic dark energy models is as a generalization of Heisenberg's uncertainty principle.

Early applications of this principle concerned mainly black-hole thermodynamics [9–11], but more recently also cosmological topics, such as inflation [12,13], non-singular universe construction [14,15] and the dark energy model [16,17]. The underlying idea is that a generalization of the principle can modify the entropy-area relation in thermodynamics, thereby introducing corrections to the cosmological evolution equation. One well-known example is the "generalized uncertainty principle" (GUP) [18], $\Delta x\Delta p \geq 1/2 + \alpha l_p^2/2\Delta p^2$, which allows the introduction of quantum-gravity into ordinary quantum mechanics via the deformation of the Heisenberg uncertainty principle. Such a deformation implies the existence of a minimum length, $\Delta x_{\min} \sim \alpha^{1/2} l_p$, and is expected to have been most apparent in the early universe or in the high-energy regime. Another possible generalization is the "extended uncertainty principle" (EUP) [19], $\Delta x\Delta p \geq 1/2 + \beta/L_x^2\Delta x^2$, where β is a dimensionless parameter and L_x is an unknown fundamental length scale. In contrast to the GUP, the EUP implies the existence of a minimum momentum with positive values of β, $\Delta p_{\min} \sim \beta^{1/2}/L_x$, and is predicted to be most apparent at later times in the universe. However, as mentioned in [20], it is interesting that a positive cosmological constant can only result from negative values of β, thus contradicting the EUP prediction of a minimum momentum. In such a case, a position measurement may not exceed an unknown length scale, i.e., the maximum length $\Delta x_{\max} \sim L_x/\beta^{1/2}$. By combining the EUP and GUP (GEUP) we obtain a more general form [18,21]

$$\Delta x\Delta p \geq \frac{1}{2}\left(1 + \alpha l_p^2\Delta p^2 + \frac{\beta}{L_x^2}\Delta x^2\right). \quad (1)$$

This formulation of the GEUP (1) predicts the existence of both a minimum and a maximum length (with negative values of β). It is worth mentioning that these modified Heisenberg uncertainty principles (i.e., the GUP, EUP, or GEUP), can yield a correction to the Bekenstein–Hawking entropy of a black hole [22,23].

The connection between thermodynamics and gravity was first investigated by Bardden, Carter, and Hawking [24]. There has since then been an abundant literature on, e.g., the Rindler space–time [25] and the Friedmann–Robertson–Walker (FRW) universe [26]. With regard to the Rindler space–time, Jacobson found that the Einstein equation can be derived from the thermodynamic relation between heat, entropy, and temperature: $dQ =$

E-mail address: seyen@skku.edu.

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TdS, where dQ is the energy flux and T is the Unruh temperature, which are detected by an accelerated observer located just within the local Rindler causal horizons. The FRW universe, on the other hand, assumes that the apparent horizon $\tilde{r}_A$ has an associated entropy $S = A/4G$ and a temperature $T = \kappa/2\pi$ in Einstein gravity, where A and κ are, respectively, the area and surface gravity of the apparent horizon. Akbar and Cai derived the differential form of the Friedmann equation for a FRW universe from the first law of thermodynamics at the apparent horizon, i.e., $dE = TdS + WdV$, where E is the total energy density of matter existing within the apparent horizon, V is the volume contained within the apparent horizon, and the work density $W = (\rho - p)/2$ is a function of the energy density ρ and the pressure p of matter in the universe. A modified Friedmann equation was recently suggested [27] based on a corrected entropy formula that is potentially useful within the context of cosmology.

The purpose of the present study is to consider cosmology within the framework of the GEUP, i.e., by considering the minimum and maximum lengths, and to compare the results with observations. The GEUP involves two parameters that can be constrained by measurements.

This paper is organized as follows: In Section 2, we investigate the influence of the GEUP on thermodynamics, and obtain a corrected Friedmann equation for the FRW universe. In Section 3, we investigate the effects of the GEUP length-scale parameters α and β, and find that the theory is consistent with the long acceleration phase currently undergone by the universe. Section 4 presents observational constraints on our model parameters. Section 5 closes with discussions and concluding remarks.

2. Modified Friedmann equations

This section presents calculations of the modified Friedmann equations within the framework of the GEUP to describe cosmological effects. The outcome of the GEUP (1) is the modified momentum uncertainty

$$\Delta p \geq \frac{\Delta x}{\alpha l_p^2}\left(1 - \sqrt{1 - \frac{l_p^2}{L_x^2}\alpha\beta - \frac{\alpha l_p^2}{\Delta x^2}}\right), \simeq \frac{1}{2\Delta x}\left(1 + \frac{\alpha l_p^2}{4\Delta x^2} + \frac{\beta}{L_x^2}\Delta x^2\right), \quad (3)$$

with the Taylor expansion calculated at $\alpha = \beta = 0$. As noted in [22], the Heisenberg uncertainty principle $\Delta p > 1/\Delta x$ can be rewritten in terms of a lower bound to the energy ($E > 1/\Delta x$), which in the case of the GEUP becomes

$$E \geq \frac{1}{2\Delta x}\left(1 + \frac{\alpha l_p^2}{4\Delta x^2} + \frac{\beta}{L_x^2}\Delta x^2\right). \quad (4)$$

When a black hole absorbs or emits a classical particle of energy E and size R, the minimal change in the surface area of the black hole is $\Delta A_{\min} \geq 8\pi l_p E R$. Arguably, the size of a quantum particle cannot be smaller than Δx [28], which would imply the existence of a finite bound $\Delta A_{\min} \geq 8\pi l_p E \Delta x$. Thus, considering the GEUP, we obtain

$$\Delta A_{\min} \geq 4\pi l_p\left(1 + \frac{\alpha l_p^2}{4\Delta x^2} + \frac{\beta}{L_x^2}\Delta x^2\right). \quad (5)$$

Δx is the position uncertainty of a photon which can be associated with the black hole radius, $\Delta x = 2r_s$, where r_s is the Schwarzschild radius. Given the surface area of the black hole, $A = 4\pi r_s^2$, the relation between A and Δx can be expressed as $\Delta x^2 = A/\pi$. Substituting this equation into (5), the minimal area change becomes

$$\Delta A_{\min} \geq 4\pi l_p \lambda\left(1 + \frac{\pi\alpha l_p^2}{4A} + \frac{\beta}{\pi L_x^2}A\right), \quad (6)$$

where λ is the calibration factor that is determined from the Bekenstein–Hawking entropy formula. The entropy of the black hole is assumed to depend on its surface area. Also, given that the entropy increases by a factor of ln 2 at least, regardless of the value of the area, we have

$$\frac{dS}{dA} = \frac{\Delta S_{\min}}{\Delta A_{\min}} = \frac{1}{4l_p^2}\left(1 + \frac{\pi\alpha l_p^2}{4A} + \frac{\beta}{\pi L_x^2}A\right)^{-1}, \quad (7)$$

where $\ln 2/\lambda = \pi$, as mentioned above. Integrating (7), the GEUP-corrected entropy is

$$S = \frac{A}{4l_p^2}\left(1 - \frac{\pi\alpha l_p^2}{4A}\ln\left(\frac{A}{4l_p^2}\right) - \frac{\beta}{2\pi L_x^2}A\right). \quad (8)$$

We note that the modified Bekenstein–Hawking entropy (8) arises from the existence of the minimum and maximum lengths.

Based on the "apparent horizon" approach [26], we derived the modified Friedmann equations with the modified entropy (8) applied to the first law of thermodynamics, $dE = TdS + WdV$. Thus, we considered that space–time geometry is characterized by the FRW metric

$$ds^2 = -dt^2 + a^2\left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega_2^2\right), \quad (9)$$

where a is a scaling factor of our universe, and the values of the spatial curvature constant $k = +1, 0,$ or -1 correspond, respectively, to a closed, flat, or open universe. Using spherical symmetry, the metric (9) can be rewritten as

$$ds^2 = h_{ab}dx^a dx^b + \tilde{r}^2 d\Omega_2^2, \quad (10)$$

where $x^0 = t, x^1 = r$ and $\tilde{r} = ar$, and the two-dimensional metric $h_{ab} = \mathrm{diag}(-1, a^2/(1 - kr^2))$. In the FRW universe, a dynamic horizon always exists because it is a local quantity of space–time, which is a marginally trapped surface with vanishing expansion. It is determined by the relation $h^{ab}\partial_a\tilde{r}\partial_b\tilde{r}$, which yields the radius of the apparent horizon

$$\tilde{r}_A^2 = \frac{1}{H^2 + k/a^2}, \quad (11)$$

where $H = \dot{a}/a$ is the Hubble parameter. By assuming that matter in the FRW universe forms a perfect fluid with four-velocity u^μ, the energy–momentum tensor can be written

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}, \quad (12)$$

where ρ is the energy density of the perfect fluid and p is its pressure. The energy conservation law, $\nabla_\mu T^{\mu\nu} = 0$, yields the continuity equation

$$\dot{\rho} + 3H(\rho + p) = 0. \quad (13)$$

According to the main results of [26,27], applying the first law of thermodynamics to the apparent horizon of the FRW universe yields the corresponding Friedmann equations

$$\frac{8\pi G}{3}\rho = -16\pi G\int\frac{S'(A)}{A^2}dA, \quad (14)$$

$$-\pi(\rho + p) = S'(A)\left(\dot{H} - \frac{k}{a^2}\right), \quad (15)$$