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Holographic entanglement entropy and cyclic cosmology

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ABSTRACT

We discuss a cyclic cosmology in which the visible universe, or introverse, is all that is accessible to an observer while the extroverse represents the total spacetime originating from the time when the dark energy began to dominate. It is argued that entanglement entropy of the introverse is the more appropriate quantity to render infinitely cyclic, rather than the entropy of the total universe. Since vanishing entanglement entropy implies disconnected spacetimes, at the turnaround when the introverse entropy is zero the disconnected extroverse can be jettisoned with impunity.

1. Introduction

Cyclic cosmology with an infinite number of cycles which include – expansion, turnaround, contraction, bounce – is an appealing theory. As emphasized first by Tolman [1,2], entropy and the second law of thermodynamics provide a significant challenge to implementing a cyclic theory.

The present approach is based on the so-called Come Back Empty (CBE) principle which chooses the turnaround as the point in the cycles where entropy is jettisoned. A successful universe must be selected with zero entropy at the turnaround to begin an adiabatic contraction to the bounce while empty of all matter including black holes. In this article we provide a better understanding of CBE based on the holographic principle [3–5]. In particular, the notion of entanglement entropy [6] and the related question of spacetime connectivity [7,8] can put the CBE model on a quantum theoretical basis.

Although the holographic principle has been well-known for many years, its relationship to cyclic cosmology has been unclear. Even how it could help reconcile general relativity with quantum mechanics is still in a period of rapid and exciting but still incomplete development. In the AdS/CFT correspondence, it has been argued that the dynamics of spacetime on the AdS side, including the connectivity of spacetime, is related to quantum entanglement of disconnected classical theories on the CFT side. This idea was introduced by Van Raamsdonk in an essay submitted to the Gravity Research Foundation. Considerations of the entropy of the universe have been discussed also in [9] and by E. Verlinde [10,11]

Before discussing the impact on cyclic cosmology which is our present subject, let us briefly discuss how this line of reasoning changes the marriage of general relativity with quantum mechanics. For several decades it had been tacitly assumed that there exists a quantum gravity theory whose classical limit is general relativity. Now it appears that whether or not such a theory exists may not be the best question to ask. Classical general relativity which describes the dynamics of spacetime may instead arise from consistency conditions on the quantum entanglement of conformal field theories which are at the holographic boundary.

Subsequent work has been mainly on time-independent spacetimes which are inadequate to describe cosmology. However, we shall show that for the time-dependent CBE model of cyclic cosmology use of holographic quantum entanglement entropy makes possible a better understanding of entropy at the turnaround.

The contents of the present article are: in Section 2 we define the introverse and extroverse; in Section 3 we describe the turnaround between expansion and contraction, and introduce entanglement entropy as a key to solving a central problem in cyclic cosmology; finally, in Section 4, there is some discussion.

2. Extroverse and introverse in expansion era

The best way to discuss the CBE model is by using the language introduced in [12] where the spacetime occupied by the universe is divided into two subregions denoted by introverse and extroverse respectively which we must define carefully because it will be the entanglement entropy between these two subregions which will play a central rôle in the cyclicity. As in [12], the present cyclic cosmology does not involve any inflationary era.

The present visible universe at $t = t_0$ is the present introverse and its comoving radius is

$$R_{int}(t_0) = 44Gly \tag{1}$$

A more useful time to consider than the present $t_0 = 13.8$Gy is the time $t = t_{DE}$ when the dark energy began to dominate over the matter component. This occurred at $t_{DE} = 9.8$Gy at which time

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the extroverse is identified with the introverse. After $t = t_{DE}$, the extroverse expands exponentially and become larger, eventually very much larger, than the introverse.

At $t = t_{DE}$

$$R_{ext}(t_{DE}) = R_{int}(t_{DE}) = 39Gly \tag{2}$$

at a time when the scale factor, normalized to $a(t_0) = 1$, was $a(t_{DE}) = 0.75$.

The extroverse expands exponentially for $t_{DE} < t < t_T$, where t_T is the turnaround time calculated to be $t_T = 1.3$Ty in [13]. The extroverse comoving radius during this expansion is given by

$$R_{ext}(t) = (39Gly) \exp\left[\frac{t - 9.8Gy}{13.8Gy}\right] \tag{3}$$

Two interesting values at $t = t_0$ and $t = t_T$ respectively are

$$R_{ext}(t_0) = 52Gly \quad \text{and} \quad R_{ext}(t_T) = 1.5 \times 10^{42} Gly \tag{4}$$

The introverse expands more gradually for $t_{DE} < t < t_T$, being limited by the speed of light in its definition. The introverse comoving radius for the expansion era is given in terms of $a(t) = \exp[(t - 13.8Gy)/(13.8Gy)]$ by

$$\begin{aligned} R_{int}(t) &= 39Gly + c\int_{t_{DE}}^{t} dt a(t)^{-1} \\ &= 39Gly + (13.8Gly)\,[1 - \exp\{-(t - 9.8Gy)/(13.8Gy)\}] \\ &\equiv 44Gly + (13.8Gly)\,[1 - \exp\{-(t - 13.8Gy)/(13.8Gy)\}] \end{aligned} \tag{5}$$

From Eq. (5) the values of $R_{inv}(t)$ at $t = t_0$ and $t = t_T$ are

$$R_{int}(t_0) = 44Gly \quad \text{and} \quad R_{int}(t_T) = 58Gly \tag{6}$$

The behaviour of $R_{ext}(t)$ and $R_{int}(t)$ calculated in (4) and (6) respectively assumes that the dark energy is accurately described by a cosmological constant. The results for $R_{ext}(t_0)$ and $R_{int}(t_0)$ show that the present extroverse is already 60% larger in volume than the introverse implying that hundreds of billions of galaxies have exited the introverse since the onset of exponential expansion.

3. The turnaround

In the previous section we used the value $t_T = 1.3$Ty without further comment. It is quite simple to re-derive that this must be the turnaround time in the CBE model as follows.

For infinite cyclicity, it is necessary to impose a matching condition on the contraction scale factor $\hat{a}(t)$ defined as

$$\hat{a}(t) \equiv \left[\frac{R_{int}(t_T)}{R_{ext}(t_T)}\right] a(t) \tag{7}$$

with the scale factor $a(t)$ of the previous expansion. Since the contraction has a radiation-dominated behaviour $\sim \sqrt{t}$ the matching is conveniently made at the onset of matter domination during expansion when $t = t_m = 47ky$ and the expansion scale factor $a(t_m)$ is

$$a(t_m) = (0.75)\left[\frac{47ky}{9.8Gy}\right]^{\frac{2}{3}} = 2.1 \times 10^{-4} \tag{8}$$

The value of $\hat{a}(t_T)$ follow from Eq. (7) to be

$$\hat{a}(t_T) = \left(\frac{R_{int}(t_T)}{a(t_T)R_{ext}(t_0)}\right) a(t_T) = \left(\frac{R_{int}(t_T)}{R_{ext}(t_0)}\right) = 1.11. \tag{9}$$

The infinite-cyclicity matching condition, which ensures that the contracting phase matches smoothly on to the radiation-dominated expansion phase as it must to allow the cycles to repeat without changing the overall scale, is

$$\hat{a}(t_m) \equiv a(t_m) \tag{10}$$

then provides the unique solution

$$t_T = (47ky)\left(\frac{1.11}{2.1 \times 10^{-4}}\right)^{\frac{1}{2}} = 1.3Ty \tag{11}$$

for the turnaround time. Cyclicity is now known to require [12] $\omega = -1$ precisely. This precise value does not require the notion of inflation but arises naturally in the contraction era leading to the bounce, just as does the flatness condition $\Omega_{Total} = 1$. The requirement $\omega = -1$ corresponds to a time-independent cosmological constant whose lack of time evolution is a definite prediction of the Cyclic Cosmology.

We shall improve the original version of Comes Back Empty (CBE) with a new understanding based on quantum entanglement. The primordial fluctuations in a cyclic cosmology have been studied in [14] but these merit further study with respect to their anisotropy and the tilt of their power spectrum. Although quantum fluctuations as precursors of cosmic structure formation are well-known *e.g.* [14], to our knowledge this is the first application of quantum computation and quantum information [15], in particular quantum entanglement, to cosmology.

In the present derivation, the dark energy is described by a cosmological constant with $\omega = -1$. We use the Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} \tag{12}$$

at almost all times except *times extremely close to the turnaround or the bounce*. Since this involves modification of general relativity we take a phenomenological approach with

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3}\left[\left(1 - \frac{a(t)^2}{a(t_T)^2}\right)\left(1 - \frac{a(t_B)^2}{a(t)^2}\right)\right] \tag{13}$$

where t_T, t_B are the turnaround, bounce times respectively. In Eq. (13), the square bracket equals 1 at almost every time. We shall not employ Eq. (13) in the following and it is written only as an illustration.

Now we turn to the rôle of quantum entanglement. This begins from the holographic principle and its realization in the AdS/CFT duality. One previous use of quantum mechanics in cosmology has been the idea that quantum fluctuations of the inflaton field during inflation are amplified to seed large scale structure formation. Here we suggest that quantum entanglement and its relationship with spacetime connectivity play an comparably dramatic rôle by elucidating the CBE cyclic cosmology model at turnaround.

In the AdS/CFT correspondence, if we consider two non-interacting identical copies CFT_A and CFT_B, a state of the system can in general be written

$$|\Psi\rangle = |\Psi\rangle_A \otimes |\Psi\rangle_B. \tag{14}$$

The CFTs are each dual to separate asymptotically AdS spacetimes so that the direct product (14) is dual to two spacetimes which are disconnected.

As a quantum state we are free to consider a superposition of states. Let the energy eigenstates be $|E_k\rangle$ for one CFT and consider the quantum superposition

$$|\Psi(\beta)\rangle = \Sigma_k e^{-\frac{\beta}{2}E_k}|E_k\rangle \otimes |E_k\rangle \tag{15}$$

which can be shown, in general, to be dual to a single connected spacetime. Starting with $|\Psi\rangle$ we deduce that the density matrix for CFT_B is

$$Tr\,(|\Psi\rangle\langle\Psi|) = \Sigma_k e^{-\beta E_k}|E_k\rangle\langle E_k| = \rho_T \tag{16}$$

which is a thermal density matrix. so the quantum superposition of two disconnected spacetimes is identified with a classical connected spacetime. This leads to the fascinating idea that *classical*