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How to obtain a cosmological constant from small exotic $\mathbb{R}^4$

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ABSTRACT

In this paper we determine the cosmological constant as a topological invariant by applying certain techniques from low dimensional differential topology. We work with a small exotic R^4 which is embedded into the standard $\mathbb{R}^4$. Any exotic R^4 is a Riemannian smooth manifold with necessary non-vanishing curvature tensor. To determine the invariant part of such curvature we deal with a canonical construction of R^4 where it appears as a part of the complex surface $K3\#\overline{CP(2)}$. Such R^4's admit hyperbolic geometry. This fact simplifies significantly the calculations and enforces the rigidity of the expressions. In particular, we explain the smallness of the cosmological constant with a value consisting of a combination of (natural) topological invariant. Finally, the cosmological constant appears to be a topologically supported quantity.
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1. Introduction

One of the great mysteries in modern cosmology is the accelerated expansion of the universe as driven by dark energy. After the measurements of the Planck satellite (PLANCK) were completed, the model of a cosmological constant (CC) has been favored among other models explaining the expansion, like quintessence. In 1917, the cosmological constant Λ was introduced by Einstein (and later discarded) in his field equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \Lambda g_{\mu\nu}$$

($g_{\mu\nu}$ is a metric tensor, $R_{\mu\nu}$ the Ricci tensor and R the scalar curvature). By now it seems to be the best explanation of dark energy. However, the entire mystery of the cosmological constant lies in its extremely small value (necessarily non-zero, seen as energy density of the vacuum) which remains constant in an evolving universe and is a driving force for its accelerating expansion. These features justify the search for the very reasons explaining their occurrences, among them the understanding of the small value of the cosmological constant is particularly challenging. Our strategy in this paper is to compute the value of a cosmological constant as a topological invariant in dimension 4.

Such an attempt is far from being trivial or even recognized as possible. As a motivation to demonstrate the possibility, let us consider the trace of the Einstein's field equations

$$R = -4\Lambda$$

with a strictly negative but constant scalar curvature for a spacetime. It follows that the underlying spacetime must be a manifold of constant negative curvature or admitting an Einstein metric (as solution of $R_{\mu\nu} = \lambda g_{\mu\nu}$) with negative constant $\lambda = -\Lambda < 0$. The evolution of the cosmos from the Big Bang up to now determines a spacetime of finite volume. The interior of this finite-volume spacetime can be seen as compact manifold with negative Ricci curvature. That is why the corresponding spacetime manifold is diffeomorphic to the hyperbolic 4-manifold (see Appendix A about Mostow–Prasad rigidity and hyperbolic manifolds for this uniqueness result). By Mostow–Prasad rigidity [1,2], every hyperbolic 4-manifold with finite volume is rigid, i.e. geometrical expressions like volume, scalar curvature etc. are topological invariants. Then the discussion above indicates that Λ might be a topological invariant. In fact in this paper we show how to calculate the CC as a topological invariant based on some features of hyperbolic manifolds of dimension 3 and 4.

It is a rather well-founded and powerful approach in various branches of physics to look for the explanations of observed phenomena via underlying topological invariants. There are many examples of such invariant quantities known from particle physics to solid state physics as well from the history of physics. Let

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us mention just two recent examples, i.e. topological phases in strong electron interactions and emerging Kondo insulators as heavy fermions [3], or the search for experimental realizations of topological chiral superconductors with nontrivial Chern numbers (e.g. [4]).

The distinguished feature of differential topology of manifolds in dimension 4 is the existence of open 4-manifolds carrying a plenty of non-diffeomorphic smooth structures. In the computation of the CC value presented here, the special role is played by the topologically simplest 4-manifold, i.e. $\mathbb{R}^4$, which carries a continuum of infinitely many different smoothness structures. Each of them except one, the standard $\mathbb{R}^4$, is called *exotic* R^4. All exotic R^4 are Riemannian smooth open 4- manifolds homeomorphic to $\mathbb{R}^4$ but non-diffeomorphic to the standard smooth $\mathbb{R}^4$. The standard smoothness is distinguished by the requirement that the topological product $\mathbb{R}\times\mathbb{R}^3$ is a smooth product. There exists only one (up to diffeomorphisms) smoothing, the standard $\mathbb{R}^4$, where the product above is smooth. In the following, an exotic $\mathbb{R}^4$, presumably small if not stated differently, will be denoted as R^4.

But why are we dealing with R^4? As we mentioned already any R^4 (small or big) has necessarily non-vanishing Riemann curvature. However, the non-zero value of the curvature depends crucially on the embedding (the curvature is not a diffeomorphism invariant) of R^4. That is why our strategy is to look for natural embeddings of exotic R^4's in some manifold M^n and estimate the corresponding curvature of this R^4. This curvature depends on the embeddings in general. However, we can try to work out an invariant part of this embedded R^4. If we are lucky enough we will be able to construct the invariant part of R^4 (with respect to some natural embeddings into certain 4-manifold M^4) with the topologically protected curvature. We would expect that this curvature would reflect the realistic value of CC for some (canonical) M^4.

There are canonical 4-manifolds into which some exotic R^4 are embeddable. Here we will use the defining property of *small* exotic R^4: every small exotic R^4 is embeddable in the standard $\mathbb{R}^4$ (or in S^4). We analyze these embeddings in Sections 3 and 4. There exists a chain of 3-submanifolds of R^4 $Y_1 \to \cdots \to Y_\infty$ and the corresponding infinite chain of cobordisms

$$End(R^4) = W(Y_1, Y_2)\cup_{Y_2} W(Y_2, Y_3) \cup \cdots$$

where $W(Y_k, Y_{k+1})$ denotes the cobordism between Y_k and Y_{k+1} so that $R^4 = K\cup_{Y_1} End(R^4)$ where $\partial K = Y_1$. The $End(R^4)$ is the invariant part of the embedding $R^4 \subset \mathbb{R}^4$ mentioned above. In the first part of the paper we will show that the embedded R^4 admits a negative curvature, i.e. it is a hyperbolic 4-manifold. This follows from the fact that $Y_k, k = 1, 2, \ldots$ are 3-manifolds embedded into hyperbolic 4-cobordism $W(Y_k, Y_{k+1})$ and the curvature of Y_{k+1}, $curv(Y_{k+1})$, is determined by the curvature of Y_k:

$$curv(Y_{k+1}) = curv(Y_k)\exp(-2\theta) = \frac{1}{L^2}\exp(-2\theta)$$

where L is the invariant length of the hyperbolic structure of Y_k induced from $W(Y_k, Y_{k+1})$, i.e. $L^3 = vol(Y_{k+1})$, and θ is the topological parameter $\theta = -\frac{3}{2CS(Y_{k+1})}$. The induction over k leads to the expression for the constant curvature of the cobordism $W(Y_1, Y_\infty)$ as the function of $CS(Y_\infty)$

$$curv(W(Y_1, Y_\infty)) = \frac{1}{L^2}\exp\left(-\frac{3}{CS(Y_\infty)}\right).$$

This is precisely what we call the cosmological constant of the embedding $R^4 \subset \mathbb{R}^4$. It is the topological invariant. However, in case of the embedding into the standard $\mathbb{R}^4$, Y_∞ is a (wildly embedded) 3-sphere and thus its Chern–Simons invariant vanishes. This leads to the vanishing of the cosmological constant as far as the embedding into $\mathbb{R}^4$ is considered. We should work more globally, namely $R^4 \subset \mathbb{R}^4 \subset M^4$ and look for the suitable (still canonical) M^4 into which R^4 embeds and the corresponding cosmological constant of the cobordisms, determined by the embedding, assumes realistic value. Now the discussion is along the line of the argumentation at the beginning of the section where we considered a hyperbolic geometry on certain Einstein manifold. At first, one could think that it is not possible at all, that such miracle can happen, and one can find suitable M^4 giving the correct value of CC, and even it can, this M^4 could not be canonical. A big surprise of Section 6 is the existence of canonical M^4, which is the $K3\#\overline{\mathbb{C}P^2}$ where $K3$ is the elliptic surface $E(2)$, such that the embedding into it of certain (also canonical) small exotic R^4, generates the realistic value of CC as the curvature of the hyperbolic cobordism of the embedding. Again the curvature is constant which is supported by hyperbolic structure and thus it can be a topological invariant. In this well recognized case the boundary of the Akbulut cork of $K3\#\overline{\mathbb{C}P^2}$ lies in the compact submanifold K generating R^4. The boundary is a certain homology 3-sphere (Brieskorn sphere $\Sigma(2, 5, 7)$) which is neither topologically nor smoothly S^3, contrary to the previously considered case of the embedding into the standard $\mathbb{R}^4$, and the CS invariant of $\Sigma(2, 5, 7)$ does not vanish. Exotic R^4 as embedded in $K3\#\overline{\mathbb{C}P^2}$ lies between this Brieskorn sphere and the sum of two Poincare spheres $P\#P$. Thus, starting from the 3-sphere (wildly embedded) in K and fixing the size of S^3 to be of the Planck length, the subsequent two topology changes take place which allow for the embedding $R^4 \to K3\#\overline{\mathbb{C}P^2}$. Namely

$$S^3 \to \Sigma(2, 5, 7) \to P\#P.$$

Now the ratio of the curvature of the (wildly embedded) S^3 and the curvature of $P\#P$ is a topological invariant. Still there is a freedom to include quantum corrections to this expression. The corrections are also represented by topological invariants (Pontryagin and Euler classes of the Akbulut cork). The numerical calculations of the resulting invariant show a good agreement with the Planck result for the dark energy density. All details are presented in Section 6.

Some of the material seems to be very similar to our previous work [5]. Therefore we will comment about the differences between [5] and this work. Main idea of [5] is a new description of the inflation process by using exotic smoothness. Then, inflation as a process is generated by a change in the spatial topology. In particular we studied a model with two inflationary phases which will produce a tiny cosmological constant (CC). But the approach in the paper misses many important points: it was never shown why CC is a constant, the model uses a very special Casson handle (so that the attachment is the sum of two Poincare spheres) and it assumed the embedding of the Akbulut cork in the small exotic $\mathbb{R}^4$. With the results of this paper, these arbitrary assumptions will be no longer needed. CC is really a constant and we will present the reason for the constancy (the Mostow–Prasad rigidity of the spacetime). The model is natural, i.e. there are topological changes starting with the 3-sphere to Brieskorn sphere $\Sigma(2, 5, 7)$ and finally the change to the sum of two Poincare spheres. In contrast to [5], there is no freedom for other topology changes in this paper. Part of the previous work is the calculation of expansion factor which was identified with CC. The previous calculation depends strongly on the embedding. In this paper we will use a general approach via hyperbolic geometry which will produce a generic result identical to the previous work. Therefore, some results of this paper are similar to the previous work but obtained with different methods for a more general case. We will comment on it in the last two sections.

Secondly, we have to comment about the relation between causality and topology change in our model. As shown by Andersen and DeWitt [6] the singularities of the spatial topology change imply infinite particle and energy production under reasonable laws of quantum field propagation. Here, the concept of causal continuity is central. Causal continuity of a spacetime means,

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