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MOG without anomaly

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ABSTRACT

We obtain the action of Moffat's Modified Gravity (MOG), a scalar–tensor–vector theory of gravitation, by generalizing the Horava–Witten mechanism to fourteen dimensions. We show that the resulting theory is anomaly-free. We propose an extended version of MOG that includes fermionic fields.

1. Introduction

Moffat's proposed modified gravity theory (MOG) is a scalar–tensor–vector theory of gravitation: in addition to the tensor field of General Relativity, it also contains scalar and vector fields [1]. The two scalar fields and the massive Proca vector field of this theory add several new degrees of freedom to the system, which can help us consider the evolution of universe in new ways [2]. This theory also offers a way to investigate the origin of gravitational waves, which are emitted by merging black holes and detected by the LIGO-Virgo collaboration data [3]. The question then arises: what is the origin of this theory? Also, is this gravitational model anomaly-free? We attempt to answer these questions by using the Horava–Witten mechanism.

In 1995, Horava and Witten have shown that in eleven dimensions, all anomalies in field theory and supergravity can be canceled [4,5]. In their model, which is known as M-theory, the 10-dimensional $E_8 \times E_8$ heterotic supergravity is generalized to an 11-dimensional supergravity theory on the orbifold $R^{10} \times S^1/Z_2$ and its anomaly is canceled. In 1996, in order to solve the cosmological constant problem in four dimensions, Vafa suggested an extension of Witten's proposal to twelve dimensions. He reformulated the type IIB theory in terms of a 12-dimensional "F-theory" and showed that compactification of M-theory on a manifold "K", which admits an elliptic fibration, is equivalent to compactification of F-theory on Calabi–Yau threefolds [6]. Until 2006, the algebra that could apply in M-theory and F-theory and produce the expected actions for branes was unclear. About ten years ago, Bagger and his co-authors introduced Lie three-algebra and formulated all Lagrangians in terms of it [7–10]. However, we have no exact information about the structure of the world and its exact dimensionality. In fact, we cannot even limit it to eleven or twelve dimensions. For this reason, supergravity in twelve dimensions were considered and its solutions have been obtained [11,12]. We will generalize these mechanisms to fourteen dimensions, remove the anomalies and obtain the exact form of action for MOG.

This paper consists of two main parts. In Section 2, we show that by generalizing the Horava–Witten mechanism to fourteen dimensions, the action MOG emerges and the anomaly is removed. In Section 3, we extend this gravity theory by including a fermionic field.

2. MOG without anomaly in fourteen dimensions

Our goal is to show that by adding a 3-dimensional manifold to 11-dimensional spacetime in the Horava–Witten mechanism, all anomalies can be removed and an action without anomaly can be produced. This action is identical to the action of the modified gravity (MOG) theory presented in [1].

First, we introduce the Horava–Witten mechanism in 11-dimensional spacetime. In this model, the bosonic part of the action in 11-dimensional supergravity (SUGRA) is given by [4,5]:

$$S_{\text{Bosonic-SUGRA}} = \frac{1}{\bar{\kappa}^2}\int d^{11}x\sqrt{g}\left(-\frac{1}{2}R - \frac{1}{48}G_{IJKL}G^{IJKL}\right) + S_{\text{CGG}},$$
$$S_{\text{CGG}} = -\frac{\sqrt{2}}{3456\bar{\kappa}^2}\int_{M^{11}} d^{11}x\varepsilon^{I_1I_2\ldots I_{11}}C_{I_1I_2I_3}G_{I_4\ldots I_7}G_{I_8\ldots I_{11}}, \quad (1)$$

where $\varepsilon^{I_1I_2\ldots I_k}$ is the rank-k Levi-Civita pseudotensor and CGG is used to denote the product term of the three-form field $C_{I_1I_2I_3}$ and

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four-form field G_{IJKL}, which are directly related to the gauge field A^I, field strength F^{IJ} and Ricci curvature R^{IJ} [5]:

$$
\begin{aligned}
G_{IJKL} &= -\frac{3}{\sqrt{2}}\frac{\kappa^2}{\lambda^2}\varepsilon(x^{11})(F_{[IJ}F_{KL]} - R_{[IJ}R_{KL]}) + \dots, \\
\delta C_{ABC} &= -\frac{\kappa^2}{6\sqrt{2}\lambda^2}\delta(x^{11})\,\mathrm{tr}(\epsilon_C F_{AB} - \epsilon_C R_{AB}), \\
G_{11ABC} &= (\partial_{11}C_{ABC} \pm 23 \text{ permutations of the indices 11 and } ABC) \\
&\quad + \frac{\kappa^2}{\sqrt{2}\lambda^2}\delta(x^{11})\omega_{ABC}, \\
\delta\omega_{ABC} &= \partial_A\,\mathrm{tr}(\epsilon F_{BC}) + \text{cyclic permutations of } ABC, \\
F^{IJ} &= \partial^I A^J - \partial^J A^I, \\
R_{IJ} &= \partial_I \Gamma^B_{JB} - \partial_J \Gamma^B_{IB} + \Gamma^A_{JB}\Gamma^B_{IA} - \Gamma^A_{IB}\Gamma^B_{JA}, \\
\Gamma_{IJK} &= \partial_I g_{JK} + \partial_K g_{IJ} - \partial_J g_{IK}, \\
\hat{G}_{IJ} &= R_{IJ} - \frac{1}{2}R g_{IJ},
\end{aligned} \tag{2}
$$

where ϵ and ϵ_C characterize infinitesimal gauge transformations [5]. Here, $\varepsilon(x^{11})$ is 1 for $x^{11} > 0$ and -1 for $x^{11} < 0$ and also $\delta(x^{11}) = \frac{1}{2}\partial\varepsilon(x^{11})/\partial x^{11}$ is the Dirac delta function. As usual [5], tr is 1/30th of the trace Tr in the adjoint representation for $E_8 \times E_8$. The ellipsis $(\dots)$ denotes terms that are regular near $x^{11} = 0$ hence vanish there [5].

The gauge variation of the CGG term in the action yields the following equation [5]:

$$
\begin{aligned}
\delta S_{\mathrm{CGG}}|_{11} &= -\frac{\sqrt{2}}{3456\bar{\kappa}^2}\int_{M^{11}} d^{11}x\varepsilon^{I_1 I_2 \dots I_{11}}\delta C_{I_1 I_2 I_3} G_{I_4 \dots I_7} G_{I_8 \dots I_{11}} \\
&\approx -\frac{\bar{\kappa}^4}{128\lambda^6}\int_{M^{10}} \Sigma^5_{n=1}(\mathrm{tr}\,F^n - \mathrm{tr}\,R^n + \mathrm{tr}(F^n R^{5-n})),
\end{aligned} \tag{3}
$$

where $\mathrm{tr}\,X^n = \mathrm{tr}(X_{[I_1 I_2}..X_{I_{2n-1}I_{2n}]}) = \varepsilon^{I_1 I_2 .. I_{2n-1} I_{2n}} X_{I_1 I_2}..X_{I_{2n-1}I_{2n}}$. The above terms cancel the anomaly of $(S_{\mathrm{Bosonic-SUGRA}})$ in 11-dimensional manifolds [5]:

$$
\delta S_{\mathrm{CGG}}|_{11} = -\delta S^{\mathrm{anomaly}}_{\mathrm{Bosonic-SUGRA}}. \tag{4}
$$

Thus, S_{CGG} is necessary for anomaly cancelation. Our goal now is to find a good rationale for its inclusion. We also answer the issue of the origin of CGG terms in 11-dimensional supergravity. In fact, we propose a theory in which CGG terms appear in the supergravity action without being added by hand. To this end, we will show that first, there are only point like manifolds with scalars which attach to them. By joining these manifolds, 1-dimensional manifolds are emerged which gauge fields and gravitons live on them. Then, these manifolds glue to each other and build higher N-dimensional manifolds with various orders of gauge fields and curvatures. Gauge fields are strings with two ends which end produces one indice and totally field strength of gauge field has two indices. Some gauge fields join to each other and form G-fields. These G-fields are constructed from linking two strings and have four ends. Each end of string produce one indice and thus G-fields have four indices. By breaking one N-dimensional manifold, two lower dimensional manifolds (child manifolds) are produced which are connected by an extra manifold, called Chern–Simons manifold. Some strings are strengthed between child manifolds and produce anomaly. At this stage, one end of some G-fields is located on Chern–Simons manifold and three other ends are placed on one of child manifolds. An observer that lives on one of child manifolds sees only three ends of some of G-fields. These fields play the role of Chern–Simons fields or C-fields in supergravity. The anomaly which is produced in child manifolds can be removed by extra terms which are produced by Chern–Simons manifold and gravity can be anomaly free. In fact, if we sum over energies of child manifolds and Chern–Simons one get the energy of initial big Manifold which is anomaly free. We discuss this subject in detail.

Before discussing the process of formation of various manifolds with different dimensions, we should obtain a relation between string fields and matters and gravity. This helps us to re-formulate field theory in terms of derivatives of strings. To do this, first, we assume that our universe is born on a D3-brane. Thus, evolution of universe can be controlled by evolution of this brane and strings which live on it. Let us to introduce the action of D3-brane which is given by [13]:

$$
\begin{aligned}
S_{D3} &= -T_{D3}\int d^4y\sqrt{-det(\bar{\gamma}_{ab} + 2\pi l_s^2 F_{ab})}, \\
\bar{\gamma}_{ab} &= g_{\mu\nu}\partial_a X^\mu \partial_b X^\nu, \\
F_{ab} &= \partial_a A_b - \partial_b A_a
\end{aligned} \tag{5}
$$

where A_b is the gauge field, F_{ab} is the field strength, X^μ is the string, $g_{\mu\nu}$ is the metric, T_{D3} is tension and l_s is the string length. Substituting $X^0 = t$ and doing some mathematical calculations, the action D3-brane in Eq. (5) is given by [13]:

$$
S_{D3} = -T_{D3}\int d^4y\sqrt{1 + g_{ij}\partial_a X^i \partial^a X^j - 4\pi^2 l_s^4 F_{ab}F^{ab}}, \tag{6}
$$

To construct our universe on a D3-brane, this action should be equal to the action of fields and gravity in 4-dimensional universe. The action of matter and gravity is given by:

$$
\begin{aligned}
S_{Gravity-Matter} &= \int d^4y\sqrt{-g}\Big(R + g_{ab}\partial^a\phi\partial^b\phi - i\bar{\psi}\gamma^a\partial_a\psi \\
&\quad + A_i A^i + \frac{1}{2}\phi^2 + 1\Big),
\end{aligned} \tag{7}
$$

where ϕ is the scalar field and ψ is the fermionic field. Putting equation of (6) equal to Eq. (7) and doing some mathematical calculations, we obtain the relation between strings and matter fields: (see equation given in Box I)

where we have assumed $T_{D3} \simeq 1$ and $4\pi^2 l_s^4 \simeq 1$. This equation shows that there is direct relation between strings and fields in 4-dimensional field theory. Using this relation, we can consider the process of formation of manifolds and fields which live on them.

At this stage, we will show that at the beginning, there are point like manifolds in space (see Fig. 1) which strings are attached to them. These manifolds have only one dimension in direction of time. All interactions between strings on one manifold are the same and are concentrated on one point which manifold is located on it. The potential of these interactions can be shown by a delta function and thus, the energy of manifold tends to one.

$$
\begin{aligned}
& V(\tilde{X}^I) = \delta(\tilde{X}^I) \\
& E_{M^0} = 1 = \int_{M^0} d\tilde{X}^I V(\tilde{X}^I) = \int_{M^0} d\tilde{X}^I \delta(\tilde{X}^I) = \\
& \int_{M^0} d\tilde{X}^I\Big(\frac{1}{\sqrt{2\pi y}}e^{-\frac{\tilde{X}^I\tilde{X}_I}{2y}}\Big)
\end{aligned} \tag{9}
$$

where M^0 denotes the point like manifold, $\tilde{X}^I$'s are strings which attached to them and y is the length of point which shrinks to zero. With new redefinition of string fields $\tilde{X}^I \longrightarrow \sqrt{2\pi y}X^I$, we get:

$$
E_{M^0} = 1 = \int_{M^0} dX^I e^{-\pi X^I X_I} \tag{10}
$$

We can calculate the integral and obtain a solution for strings (X^I):

$$
\begin{aligned}
& \int_{M^0} dX^I e^{-\pi X^I X_I} = 1 \longrightarrow \\
& \frac{1}{2}erf(\sqrt{X^I X_I \pi}) = 1 \longrightarrow \\
& X^I \approx \epsilon^J I
\end{aligned} \tag{11}
$$