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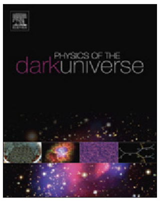

Uncertainties in primordial black-hole constraints on the primordial power spectrum

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ABSTRACT

The existence (and abundance) of primordial black holes (PBHs) is governed by the power spectrum of primordial perturbations generated during inflation. So far no PBHs have been observed, and instead, increasingly stringent bounds on their existence at different scales have been obtained. Up until recently, this has been exploited in attempts to constrain parts of the inflationary power spectrum that are unconstrained by cosmological observations. We first point out that the simple translation of the PBH non-observation bounds into constraints on the primordial power spectrum is inaccurate as it fails to include realistic aspects of PBH formation and evolution. We then demonstrate, by studying two examples of uncertainties from the effects of critical and non-spherical collapse, that even though they may seem small, they have important implications for the usefulness of the constraints. In particular, we point out that the uncertainty induced by non-spherical collapse may be much larger than the difference between particular bounds from PBH non-observations and the general maximum cap stemming from the condition $\Omega \leq 1$ on the dark-matter density in the form of PBHs. We therefore make the cautious suggestion of applying only the overall maximum dark-matter constraint to models of early Universe, as this requirement seems to currently provide a more reliable constraint, which better reflects our current lack of detailed knowledge of PBH formation. These, and other effects, such as merging, clustering and accretion, may also loosen constraints from non-observations of other primordial compact objects such as ultra-compact minihalos of dark matter.

1. Introduction

Cosmological observations, in particular those of the cosmic microwave background (CMB) anisotropies [1,2], place tight constraints on the properties of the primordial density (or curvature) fluctuations on large scales through the accurate measurements of the primordial power spectrum [1,3]. These measurements, however, probe only a relatively small range of scales, ie., wave numbers between $k \sim 10^{-3}\,\mathrm{Mpc}^{-1}$ and $k \sim 1\,\mathrm{Mpc}^{-1}$. Even though the measured power spectrum on these cosmological scales provides strong evidence in support of an inflationary phase [4–7] in the early Universe, and constrains various inflationary models and their parameters [3], it only probes a small region of the inflaton potential. It has therefore been of importance to try to extend the constraints on the curvature power spectrum to a wider range of scales using other cosmological and astrophysical measurements. Power-spectrum constraints have been extended to $k \sim 10^4\,\mathrm{Mpc}^{-1}$ through measurements of the CMB spectral distortions [8,9], and to $k \sim 10^4\,\mathrm{Mpc}^{-1}$ using constraints on entropy production between Big Bang nucleosynthesis and today [10], although these are currently only (fairly weak) upper bounds on the amplitude of the spectrum.

One important set of such additional constraints on small scales has been provided by non-observations of primordial black holes (PBHs) [11,12] that are expected to have formed in the early Universe when very large density perturbations collapsed. In most scenarios, these overdensities are of inflationary origin [13–15].[1] As soon as the overdense regions come back into causal contact after inflation, they collapse if they exceed a medium-specific threshold. PBHs form mainly in the radiation-dominated epoch, hence a radiation medium is considered. Because of the connection between the formation of PBHs and the amplitude of primordial fluctuations, observations or non-observations of PBHs could potentially

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[1] Many more possibilities for PBH formation exist, and we refer the interested reader to corresponding reviews (e.g. Ref. [16]).

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further constrain inflationary dynamics on scales far smaller than the cosmological ones. PBHs have not been observed yet, and constraints from their non-observations have been used [14,17–21] to place (currently weak) upper limits on the amplitude of the power spectrum over a very wide range of scales (from $k \sim 10^{-2}$ Mpc^{-1} to $k \sim 10^{23}$ Mpc^{-1} [14]). Similar arguments are behind another type of constraints on the power spectrum at small scales, namely those from the non-observations of ultra-compact minihalos of dark matter (UCMHs) [22,23] that are expected to have formed shortly after matter-radiation equality when a perturbation with a very large amplitude but not large enough to collapse to a black hole enters the horizon [24–26].

By using well-known effects, in this *Letter* we question the current use of constraints on the existence and abundance of PBHs (and similarly of UCMHs) to constrain the primordial power spectrum. We first demonstrate, via a simple example, that taking into account the known realistic aspects of the PBH (and UCMH) formation and evolution, such as the well-studied critical collapse, makes the simple translation of the constraints on their existence and abundance into constraints on the power spectrum inaccurate. This, in turn, implies that the local details of the current non-observation constraints are not reliable, as the critical-collapse effect washes them out. We then demonstrate that the effect of non-spherical collapse, as is already known, can induce uncertainties that are orders of magnitudes larger than the differences between direct constraints from non-observations of PBHs and the overall limit from the maximum total dark matter. This renders the former constraints practically useless. We therefore suggest the early-Universe model-builders to apply only this overall maximum dark-matter constraint to their models, as it currently provides the most reliable constraint on the primordial power spectrum from PBHs.

2. Primordial black holes and power-spectrum constraints

The naïve first estimate of PBH formation postulates that the holes formed this way would have a mass M of the order of the mass $M_{\rm H}$ of a black hole of horizon size equal to the Universe's horizon at their time of formation. If the underlying primordial power spectrum is Gaussian, the simplest assumptions dictate that the fraction β of the collapsed patches in the Universe at their time of formation is given by [27]

$$\beta \approx \mathrm{Erfc}\left(\frac{\delta_c}{\sqrt{2}\,\sigma}\right). \tag{1}$$

Here, δ_c is the critical overdensity, which, in radiation domination, is found to be approximately equal to 0.45 (cf. Ref. [28]). Note that this value is essentially independent of the mass of the collapsing space–time region. In Eq. (1), σ denotes the root-mean square of the primordial density power spectrum P_δ. Erfc is the complementary error function Erfc $\equiv$ 1 − Erf, with Erf being the standard error function. For values of β between 0 and 2, this function is invertible, and hence it might appear reasonable that a constraint on the density of PBHs of a certain mass could be translated into a constraint on the primordial power spectrum (as has been, for instance, performed in Refs. [18–21]).[2] This procedure would open up a possible way to constrain the primordial power spectrum at much smaller scales than those accessible with CMB observations [1,3].

However, this naïve procedure has been shown to be too simplistic and may lead to tremendous errors (cf. Refs. [29,30]). There are several important effects which need to be accounted for in order for the transformation between the primordial power spectrum and the PBH mass distribution to yield reliable answers.

3. Sources of uncertainty

The first and best studied of these effects is perhaps that of the critical collapse. As has been argued theoretically [31,32], and was later also found in numerical investigations [27,28,33,34] of collapse in full general relativity, primordial black holes actually form through the so-called *critical collapse*. By this, it is meant that the holes are not produced mono-chromatically, with their mass M just being equal (or proportional) to the horizon mass, but form subject to the so-called *critical scaling*

$$M = k M_{\rm H} (\delta - \delta_c)^{\gamma}. \tag{2}$$

Here, k is a real, positive constant, and the quantity δ denotes the overdensity. In radiation domination and for spherical density profiles — which will be assumed from now on — one finds $\gamma = 0.36$ and $k = 3.3$ (cf. Ref. [28]). Eq. (2) describes the generation of a PBH mass distribution at each instance of time at which the overdensities reenter the horizon. Generically, also the initial spectrum of overdensities will be extended, leading to PBH formation in a range of different horizon masses, which will then be convoluted with the critical-collapse effect. Hence, *one looses the one-to-one correspondence between PBH mass and formation time/scale.*

If the shapes of the overdensities are non-spherical, which is to be expected in a realistic distribution, this may also lead to large effects, as has been pointed out recently in Ref. [30]. This non-sphericity effect is likely to strongly reduce the overall production of PBHs, yielding significantly weaker constraints on the primordial power spectrum from PBH non-observations. Although corresponding detailed numerical studies are still lacking, utilizing the estimate for ellipsoidal collapse of Ref. [30], we are essentially led to changes in the threshold of the density contrast δ_c, which increases as

$$\delta_c \rightarrow \delta_{ec} \equiv \delta_c \left[1 + \kappa \left(\frac{\sigma^2}{\delta_c^2} \right)^{\nu} \right]. \tag{3}$$

This is exactly the functional form as found in the study of ellipsoidal galactic halo formation [35]. The parameters κ and ν are unknown, but suggestions can be the theoretical naïve estimates $\kappa = 9/\sqrt{10\pi}$ and $\nu = 1/2$, or halo-like estimates $\kappa = 0.47$ and $\nu = 0.67$. It is easy to show, by demanding the same final value of β (cf. Eq. (1)), and given a model with a certain threshold δ_c and a variance σ_1, that changing to a new threshold δ_{ec} (cf. Eq. (3)) demands the replacement

$$\sigma_1 \rightarrow \sigma_2 \approx \frac{\delta_c}{\mathrm{W}\left(\exp\left[-\delta_c / (2\,\sigma_1^2) \right] \delta_{ec}^2 / \sigma_1^2 \right)}. \tag{4}$$

Here, W is the Lambert W-function, which is defined as the inverse function of $f(\mathrm{W}) = \mathrm{W}\exp(\mathrm{W})$.

Another source of potential uncertainty is provided by primordial non-Gaussianities. These might be particularly relevant as PBH formation requires density contrasts of large, ie. $\mathcal{O}(1)$, magnitude deep inside the tail of the respective distribution where deviations from the Gaussian spectrum may have a particularly large effect. Like non-sphericities, these more or less shift the PBH abundance, albeit this effect can be in both directions, depending on the sign of the non-Gaussianity [16,36–38]. However, if a significant amount of dark matter exists in the form of PBHs formed from a non-Gaussian spectrum, this will also add isocurvature effects to the CMB. As these are strongly constrained, PBHs from such an origin should be ruled out *a priori* as a large contributor to dark matter;

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[2] In the case of a non-Gaussian power spectrum, the functional form for β is modified, but the function is still invertible over the same domain.