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Large distance modification of Newtonian potential and structure formation in universe

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ABSTRACT

In this paper, we study the effects of super-light brane world perturbative modes on structure formation in our universe. As these modes modify the large distance behavior of Newtonian potential, they effect the clustering of a system of galaxies. So, we explicitly calculate the clustering of galaxies interacting through such a modified Newtonian potential. We use a suitable approximation for analyzing this system of galaxies, and discuss the validity of such approximations. We observe that such corrections also modify the virial theorem for such a system of galaxies.

1. Introduction

We approximate the galaxies as point particles, and analyze the clustering of a system of such galaxies. This approximation will be valid as the distance between two galaxies is many orders of magnitude larger than the size of a single galaxy. Thus, we use techniques of standard statistical mechanical to analyze the clustering of a system of galaxies. It may be noted that such an analysis has already been performed using the usual Newtonian potential [1–5], and thus the techniques of statistical mechanics has already been used to analyze the clustering of galaxies. However, we have to either consider dark matter, or a modified Newtonian potential to explain the physics at large scales, we will analyze the clustering of galaxies using a Newtonian potential modified by super-light modes of a brane world model. We would like to point out that in this formalism a cosmic energy equation for a system of galaxies was obtained using the standard techniques of statistical mechanics [6,7]. This was used to analyze the clustering of a system of galaxies using correlation functions [8–10]. In this formalism, the correlation function and the power spectrum characterize the distribution of galaxies in clusters and superclusters [11].

So, in this paper, we analyze the clustering of galaxies using this formalism. In fact, we will use the large distance corrections to the Newtonian potential from super-light modes in brane world models [12]. These cosmological models have been motivated from string theory due to extra dimensions in string theory [13]. In these brane world models, our universe is a brane in a higher dimensional bulk. These models have been used for resolving the hierarchy problem and the weakness of gravitational force in comparison with other three fundamental forces [14]. In fact, even though there are different models for brane world theories [15], a common feature of all of these different models is that the standard model fields are confined to the four dimensional brane and the gravitons propagate into the bulk confined to the brane and thus can propagate into the higher dimensional bulk [16–18]. Due to the propagation of gravity into higher dimensions, the Newtonian potential gets brane corrections. Furthermore, as the general relativity along with its Newtonian approximation have not been tested at very large or very small distances, it is possible that the Newtonian potential would get modified at such distances. Generally, Newtonian potential may be modified due to several effects like dark energy [19,20]. So, usually, the corrections generated from brane world gravity modify the Newtonian potential at small distances [13,21], and these modifications cannot produce any new astrophysical or cosmological effects. However, it is possible to obtain super-light perturbation modes in brane world models, and these super-light modes can modify gravity at large scales [12,22].

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The importance of the models with super-light perturbation modes is due to the fact that these predict a modification of the gravitational interaction for matter on the brane at astronomical scales. The corrections to Newton's gravity due to such consideration may be promising for resolving the issue of dark matter in galaxies and galaxy clusters and even the cosmological dark energy problem [12]. The form of corrected Newtonian potential is given as [23]

$$\phi = \phi_N \left(1 + \frac{k}{r^2}\right), \quad (1)$$

where ϕ_N is the standard Newtonian potential given by,

$$\phi_N = -\frac{Gm^2}{r}. \quad (2)$$

It may be noted that this long distance correction scales as $1/r^3$, which is unlike the short distance correction which scales as $1/r^2$ [12]. As this correction changes the Newtonian potential at large distances, this can be used in analyzing the dynamics of galaxies [24–27]. In fact, it has been demonstrated that the brane world models can explain the rotation curve of galaxies better than the models which are based on the existence of dark matter [28].

It may be pointed out that phenomenologically motivated modified theories of gravity (MOG) [29,30] have been used as an alternative to dark matter. In fact, modified Newtonian dynamics (MOND) [31] and MOG [32,33], as two possible modified theories of gravity have been used to obtain the correct rotation curves of galaxies. The MOG modifies the large distance behavior of Newtonian potential [32,33], and this modification produces the correct rotation curve of galaxies. Thus, it is important to consider large distance correction to Newtonian potential for analyzing astrophysical phenomena. An advantage of using the corrections from super-light perturbation modes in brane world models is that such corrections are motivated from theoretical considerations and cosmological models motivated from string theory, but they can also have interesting phenomenologically applications [24–27]. In this paper, we will use this long distance correction to the Newtonian potential produced by super-light modes, and analyze its effect on the clustering of galaxies.

Moreover, one may note that even though there are problems with certain distance based modifications of gravity, such as MOND, in order to explain the clustering of galaxies [34–36], it has been argued that other kind of modifications to gravity can explain clustering of galaxies [37,38]. In fact, it is possible to modify MOND in such a way, that force law approximates MOND at large and intermediate accelerations, and gets further modified at ultra-low accelerations. Such ultra-low accelerations are relevant to the galaxy clusters, and such a modification has been observed to be consistent with the observations [39–42]. It has been demonstrated that MOG, which modifies the Newtonian law of gravitation, can consistently explain the clustering of galaxies [43–45]. It is also possible to explain the clustering of galaxies without dark matter by using a modified theory of gravity based on covariant Galileon model [46]. So, even though the modification of gravity such as MOND cannot be used to analyze the clustering of galaxies, it is possible to have alternative theories of gravity, which may explain such a model.

So, even though there are problems with MOND in explaining clustering of galaxies, it is possible to consider other models of modified gravity, which do not have above discussed problems. Furthermore, as the clustering of galaxies has been studied using techniques of statistical mechanics with Newtonian potential [47–52], it would be both important and interesting to generalize such an analysis of modified law of Newtonian gravity. Even though there might still be problems with such an approach, it would be a better approximation to explain the clustering of galaxies. We could improve this analysis further by incorporating dark matter, but the use of modified Newtonian potential would produce better results than the standard Newtonian potential. It has been argued that the modified theories of gravity are produced from dark matter models [53–56]. It has also been discussed that clustering can be explained using brane world models [25,57–60]. So, this motives us to use techniques of statistical mechanics [47–52], with brane world modified Newtonian potential, to galactic clustering.

2. Clustering parameter

In this section, we review the clustering of galaxy and exact equation of states in brane world corrected Newtonian potential [61]. It is possible to consider super-light modes in a brane world models, and they modify the large distance behavior of the Newton's law as

$$\Phi_{i,j} = -\frac{Gm^2}{(r_{ij}^2+\epsilon^2)^{1/2}}\left(1 + \frac{k_l}{(r_{ij}^2+\epsilon^2)}\right), \quad (3)$$

where relative position vector (between i and j particles) is $r_{ij} \gg \Lambda = |k_l|^{1/2}$, where Λ is considered as a typical length scale at which correction due to these super-light modes becomes dominant. The parameter ϵ is a regularization parameter, which occurs due to the extended structure of galaxies. The reason for considering extended structure is as following. It is clear from expression (1) that the potential energy diverges for the point-mass (i.e., $r = 0$) nature of galaxies. This will lead to a divergence in the Hamiltonian and, consequently, to the partition function. This divergence can be removed by considering extended nature of galaxies (galaxies with halos) with the help of the softening parameter ϵ, which assures that the galaxies are of finite size [2,62]. The typical range of the softening parameter is $0.01 \le \epsilon \le 0.05$ in units of the constant cell. It may be noted that at small enough distances this modified Newtonian potential reduces to the usual Newtonian potential. This is the limit in which the contribution from these super-light modes can be neglected. Furthermore, this is required from the physical constraints, as the Newtonian limit of general relativity has been well tested at such scales. Now it is possible to obtain the two-particle function form this modified potential as

$$f_{i,j} = \exp\left[\frac{Gm^2}{T(r_{ij}^2+\epsilon^2)^{1/2}}\left(1 + \frac{k_l}{(r_{ij}^2+\epsilon^2)}\right)\right] - 1. \quad (4)$$

This will further lead to the modification of configurational integrals Q_N. For instance, the configurational integral for $N = 1$, $Q_1(T,V) = V$, and (for large r where the higher terms of $\frac{k_l}{(r^2+\epsilon^2)^{1/2}}$ can be neglected) the configurational integral for $N = 2$,

$$Q_2(T,V) = 4\pi V \int_0^{R_1}\left[r^2 + \left(\frac{Gm^2}{T}\right)\frac{r^2}{(r^2+\epsilon^2)^{1/2}} \times \left(1 + \frac{k_l}{(r^2+\epsilon^2)}\right)\right] dr. \quad (5)$$

Evaluating the integrals, we obtain

$$Q_2(T,V) = V^2\left(1 + \alpha_1 x + \alpha_2 x\right), \quad (6)$$

where $x = \frac{3}{2}G^3 m^6 \bar{\rho} T^{-3}$ and

$$\alpha_1 = \sqrt{1 + \frac{\epsilon^2}{R_1^2}} + \frac{\epsilon^2}{R_1^2}\log\frac{\epsilon}{R_1 + \sqrt{R_1^2+\epsilon^2}}, \quad (7)$$

$$\alpha_2 = -2\frac{k_l}{R_1\sqrt{R_1^2+\epsilon^2}} + 2\frac{k_l}{R_1^2}\log\frac{R_1 + \sqrt{R_1^2+\epsilon^2}}{\epsilon}. \quad (8)$$