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Detecting the gravitational wave background from primordial black hole dark matter

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ABSTRACT

The black hole merging rates inferred after the gravitational-wave detections by Advanced LIGO/VIRGO and the relatively high mass of the progenitors are consistent with models of dark matter made of massive primordial black holes (PBH). PBH binaries emit gravitational waves in a broad range of frequencies that will be probed by future space interferometers (LISA) and pulsar timing arrays (PTA). The amplitude of the stochastic gravitational-wave background expected for PBH dark matter is calculated taking into account various effects such as initial eccentricity of binaries, PBH velocities, mass distribution and clustering. It allows a detection by the LISA space interferometer, and possibly by the PTA of the SKA radio-telescope. Interestingly, one can distinguish this background from the one of non-primordial massive binaries through a specific frequency dependence, resulting from the maximal impact parameter of binaries formed by PBH capture, depending on the PBH velocity distribution and their clustering properties. Moreover, we find that the gravitational wave spectrum is boosted by the width of PBH mass distribution, compared with that of the monochromatic spectrum. The current PTA constraints already rule out broad-mass PBH models covering more than six decades of masses, but evading the microlensing and CMB constraints because black holes appear spatially distributed in clusters.

1. Introduction

The recent detection by Advanced LIGO of the gravitational waves (GW) emitted by the coalescence of black hole binaries [1,2] and the unexpected high mass of the involved BH (36, 29, 14 and 8 solar masses $M_\odot$) have revived the interest for models of primordial black holes (PBH) that could constitute a large fraction or even the totality of the Dark Matter (DM) [3–7]. But do the present astrophysical constraints allow such a population of massive black holes? Do the detected BHs have a primordial origin [8–10]? If so, is it possible that these PBH account for the totality of DM? The debate has been re-opened by the AdvLIGO discovery.

On the *pro*-side, it has been shown that consistent merging rates with the one inferred by AdvLIGO can be obtained for two simple PBH-DM models, one extrapolating the DM halo mass function towards small scales [4], and one where PBH are regrouped in dense sub-halos such as the ultra-faint dwarf galaxies [5]. The latter model would also provide a natural explanation to the *missing satellite* and *too-big-to-fail* problems if the existence of thousands of such ultra-faint satellite galaxies were confirmed, in which the PBH population could prevent the formation of stars and make their trajectory unstable in such an environment. Furthermore it has been shown that PBH-DM halos could explain some unexpected fluctuations in the near-IR cosmic infrared background (CIB), found to be coherent with the unresolved soft X-ray background [11].

On the *con*-side, it has been proposed that BHs as massive as the ones detected by AdvLIGO could also result from a particular stellar evolution in low-metallicity environments [12]. Moreover a population of massive PBH accounting for the totality of DM could have induced signatures in the CMB anisotropy angular power spectrum[1] [13,14], or should have been detected through microlensing events of stars in the Magellanic clouds [15–17], although it is debated and model dependent [18,6]. But the physical processes leading to signatures in the CMB are subject to large uncertainties. Especially the accuracy of the Bondi accretion approximation is not well established and could be suppressed by the large BH velocities in the case of early clustering. Recently it has been claimed that the crucial mass window between 10 and

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[1] CMB spectral distortions are also expected and Ref. [13] claimed that masses larger than $M_\odot$ are ruled out by FIRAS. However Eq. (44) in [13] has a factor $(1+z)^{-2}$ missing, which leads to much less stringent constraints (by a factor of a few millions). There is now an agreement in the community that PBH-DM is not constrained by FIRAS.

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$100M_\odot$ was closed by the observations of the central star cluster within the Eridanius II dwarf galaxy [19,20]. However, the stability of the stellar cluster in Eridanus II seems to suggest that there is an IMBH at its center, which would soften these bounds [21] and make a broad mass spectrum model compatible with observations.

Finally, regarding the microlensing constraints from EROS, MACHOS and Kepler, they are naturally evaded if the PBH are clustered in the galactic halo so that the probability of finding such a cluster in the line-of-sight of the Magellanic clouds were the observations were done is actually very low [5]. With a similar argument, one can evade constraints on masses larger than $100M_\odot$ from the non-disruption of wide binaries, because the probability of disruption is suppressed by the probability of encounter with a PBH cluster.

As a consequence, further observations will be required to distinguish between the different models and in order to validate or definitively rule out the massive PBH-DM hypothesis. Future observations of numerous merging events, possibly involving even more massive BHs, or binaries with high orbital eccentricities, would allow to distinguish between a stellar or a primordial origin [22,10]. They could also allow to reconstruct the PBH mass spectrum and reveal how clustered PBH are [5], which would help to reveal their mechanism of formation in the early Universe [23,24]. The next runs of LIGO/VIRGO could increase dramatically the number of detected PBH mergers. Another rich source of information could come from the detection of (or from the constraints on) a stochastic background of gravitational waves induced by a huge population of massive PBH [25–28].

The goal of this paper is to compute the expected stochastic background of GWs produced by massive PBH clustered in compact halos and with some mass distribution, as in Refs. [5,3,23,24,29,30]. We examine more particularly how the GW background produced by primordial BH binaries could be distinguished from the one of BH stellar binaries, due to the limited impact factor in the process of PBH capture that affects the GW spectrum at frequencies probed by future space interferometers and pulsar timing arrays (PTA). Our analysis also considers the effects of eccentricities and velocity distributions, as well as the case PBH have a broad mass spectrum, instead of having all the same mass as in the recent analysis of [4,6,22,26,28]. Furthermore we discuss the detectability of the signal, not only with future Earth-based GW detectors like KAGRA [31] and ET [2] [32,33], but also with future GW detectors in space such as LISA[3] [34,35], DECIGO [36] and BBO [37], as well as with current and future limits from pulsar timing arrays such as EPTA[4] [38,39], IPTA [40] and SKA[5] [41,42].

Our main result is that the GW background induced by PBH-DM models consistent with AdvLIGO rates is detectable by LISA, and possibly by the SKA-PTA if their mass distribution is sufficiently broad. Indeed a broad mass spectrum is found to strongly boost the stochastic GW amplitude, so that the current PTA already constrain broad-mass PBH-DM models with $10^{-2}M_\odot \lesssim m_{\rm PBH} \lesssim 10^4 M_\odot$. Moreover, we find that the GW spectrum exhibits a very specific frequency dependence that will help to distinguish it from the one of non-primordial BH binaries. This effect is directly linked to the PBH clustering, enhancing their velocities and reducing the maximal impact parameter in the PBH capture process. This implies a minimal frequency for the GW emitted by binaries formed through PBH capture. Typically, for velocities larger than tens of km/s, e.g. corresponding to PBH clustering within ultra-faint dwarf satellite galaxies, we predict a suppressed spectrum at PTA frequencies and a slight but detectable modification w.r.t. the typical $f^{-2/3}$ frequency dependence of the strain $h_c(f)$, on frequencies probed by LISA.

The paper is organized as follows: The formalism used to calculate stochastic backgrounds of gravitational waves is summarized in Section 2. In Section 3 we describe our synthetic population of PBH binaries. The effect of initial eccentricity is calculated in Section 4. Section 5 focuses on PBH merging rates in both the monochromatic approximation and more realistic broad mass spectrum case. In Section 6 we compute the stochastic GW background for our model and discuss its detectability with future GW experiments and PTAs. Our results are summarized and some perspectives are presented in Section 7.

2. Stochastic backgrounds of gravitational waves

A population of massive BH binaries experiencing merging induces a stochastic background of gravitational waves (GW), that is characterized by the relative GW energy density to the critical density ρ_c today, per unit interval of logarithmic frequency,

$$\Omega_{\rm gw}(f) \equiv \frac{1}{\rho_c}\frac{{\rm d}\rho_{\rm gw}(f)}{{\rm d}\ln f}\,, \tag{1}$$

where f is the observed GW frequency. Another key quantity is the GW strain amplitude h_c, given by [43,44]

$$h_c^2(f) = \frac{4G}{\pi f^2}\frac{{\rm d}\rho_{\rm gw}(f)}{{\rm d}\ln f}. \tag{2}$$

The GW energy spectrum is given by the superposition of the GW radiation coming from merging BH binaries over the whole cosmic history,

$$\frac{{\rm d}\rho_{\rm gw}(f)}{{\rm d}\ln f} = \int_0^\infty \frac{{\rm d}z}{1+z}\,\frac{{\rm d}n}{{\rm d}z}\,\frac{{\rm d}E_{\rm gw}}{c^2{\rm d}\ln f_{\rm r}}, \tag{3}$$

where ${\rm d}E_{\rm gw}/{\rm d}\ln f_{\rm r}$ is the released energy by the merging events, per logarithmic interval of the rest-frame frequency $f_r = f(1+z)$. In the Newtonian limit, it is well approximated by

$$\frac{{\rm d}E_{\rm gw}}{c^2{\rm d}\ln f_{\rm r}} = \frac{\pi^{2/3}}{3c^2}\mathcal{M}_c^{5/3}(Gf_{\rm r})^{2/3}F(e) \tag{4}$$

$$F(e) = \left(1-e^2\right)^{-7/2}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)\,, \tag{5}$$

within the range $f_{\rm min} < f_r < f_{\rm ISCO}$, where e is the eccentricity of the orbit and $\mathcal{M}_c$ is the chirp mass of the BH binary, defined as $\mathcal{M}_c^{5/3} = m_{\rm A}m_{\rm B}M^{-1/3}$, with $m_{\rm A}$ and $m_{\rm B}$ the two BH masses and $M = M_{\rm tot} = m_{\rm A}+m_{\rm B}$. The value of the minimal frequency $f_{\rm min}$ is related to the initial BH separation a_0,

$$f_{\rm min} = \frac{(GM)^{1/2}}{\pi a_0^{3/2}}\,. \tag{6}$$

The maximal frequency $f_{\rm ISCO} \approx 4.4\,{\rm kHz}\,(M_\odot/M)$ corresponds to the innermost stable circular orbit. For stellar-mass BHs, this maximal frequency is well above the frequencies probed by PTAs and space interferometers, but it is within the range of AdvLIGO and other earth-based interferometers, for typical BH of several tens of solar masses. As explained later, the typical initial BH separation can range from 10^{-5} to 10^3 astronomical units (AU) for BH regrouped in $10^6 - 10^9 M_\odot$ halos, corresponding to frequencies ranging from 100 down to 10^{-11} Hz.

In the scenario considered here, BH binaries are formed when two PBH trajectories cross sufficiently close to each other so that they become bounded and start inspiralling, until they merge, which does not immediately produce circular orbits but instead highly eccentric orbits. Soon after starting spiraling around each other, the emission of GW also radiates angular momentum and

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[2] http://www.et-gw.eu/.

[3] http://sci.esa.int/lisa/.

[4] http://www.epta.eu.org.

[5] https://www.skatelescope.org.