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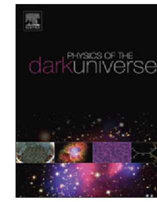

Energy scale of Lorentz violation in Rainbow Gravity

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ABSTRACT

We modify the standard relativistic dispersion relation in a way which breaks Lorentz symmetry—the effect is predicted in a high-energy regime of some modern theories of quantum gravity. We show that it is possible to realise this scenario within the framework of Rainbow Gravity which introduces two new energy-dependent functions $f_1(E)$ and $f_2(E)$ into the dispersion relation. Additionally, we assume that the gravitational constant G and the cosmological constant Λ also depend on energy E and introduce the scaling function $h(E)$ in order to express this dependence. For cosmological applications we specify the functions f_1 and f_2 in order to fit massless particles which allows us to derive modified cosmological equations. Finally, by using Hubble+SNIa+BAO(BOSS+Lyman α)+CMB data, we constrain the energy scale E_{LV} to be at least of the order of 10^{16} GeV at 1σ which is the GUT scale or even higher 10^{17} GeV at 3σ. Our claim is that this energy can be interpreted as the decoupling scale of massless particles from spacetime Lorentz violating effects.

1. Introduction

It is expected that any theory which aspires to bridge quantum theory and gravity will need to include the Planck length $\ell_P = \sqrt{\hbar G/c^3}$, where $\hbar$ is the reduced Planck constant, G is Newton's gravitational constant, and c is the speed of light. This characteristic length is derived by dimensional considerations of the constants which should appear in a regime where quantum theory, relativity, and gravity all are significant. It is expected that the Planck length is the minimum length which one can measure in a meaningful way. Associated with the Planck length is the Planck energy $E_{Pl} = \sqrt{\hbar c^5/G}$, which is simply the energy of a photon with de Broglie wavelength ℓ_P. The concept of a minimum length lies at the heart of approaches to quantum gravity such as string theory and loop quantum gravity, and has inspired a lot of theoretical work [1–6]. The idea of spacetime foam was put forth in [7] and has inspired research since then. According to this idea, quantum effects make spacetime nontrivial at small scales (the Planck scale), where particle–antiparticle pairs are continuously created and annihilated, curving spacetime at extremely small length- and time scales. This "chaotic" picture inspired the term "spacetime foam", or "quantum foam".

For some time the main approach to non-trivial spacetimes and Planck-scale effects has been Lorentz violation scenarios, which have been widely studied both theoretically and observationally. In this approach, Lorentz invariance is assumed to be broken at high energies, which introduces high-energy corrections to, for example, the dispersion relations of high-energy particles of cosmological origin. In recent years, the Rainbow Gravity framework [8] has been given a lot of attention [9–23]. This is a phenomenological approach based on Doubly Special Relativity (DSR), where the spacetime metric includes energy-dependent functions, and hence describes [24,25] universes which evolve depending on the energy of the probe particle. With the correct choice of energy dependence, problems such as singularities may be avoided in Rainbow Gravity [10]. Exploring semiclassical or phenomenological theories of quantum gravity is of vital importance to understand the low-energy quantum gravitational regime and to reach an understanding of the underlying fundamental framework.

It has been recently reported in [26] that the Rainbow framework is suitable for exploring scenarios with broken Lorentz symmetry [27–34]. In the light of this, we present the following analysis which will be concentrated on the determination of the Lorentz violation energy scale for relativistic particles by the observational data from cosmology.

This paper is organised as follows. In Section 2 we briefly outline the formalism of Rainbow Gravity and Lorentz Invariance Violation (LIV) scenarios. In Section 3 we describe the modified homogeneous Friedmann universe in the Rainbow Gravity formalism. Section 4 is dedicated to a statistical data analysis carried out which

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allows to constrain some rainbow parameters. In Section 5 we interpret our results and present some concluding remarks. Unless explicitly stated, $c = \hbar = 1$, Greek indices $\mu, \nu = 0, 1, 2, 3$, Roman indices $i, j, k = 1, 2, 3$, and the metric signature is $(-, +, +, +)$.

2. Rainbow Gravity & Doubly special relativity

The key idea of Rainbow Gravity is the modification of the spacetime metric to include energy dependent functions $f_1(E)$ and $f_2(E)$ [8], leading to a modified dispersion relation for relativistic particles of the form:

$$-E^2 f_1^2(E) + p^2 f_2^2(E) = m_0^2, \tag{1}$$

and position-space invariant of the form:

$$ds^2 = -\frac{(dx^0)^2}{f_1^2(E)} + \frac{(dx^i)^2}{f_2^2(E)}. \tag{2}$$

where m_0 is the rest mass of the particle. x^0 and x^i are the time and space coordinates, respectively. These functions are introduced by deforming the Lorentz group to include the Planck energy as a second invariant, using the formalism developed in Doubly Special Relativity (DSR) [8,24,25]. By introducing the dilatation $D = p_\mu(\partial/\partial p_\mu)$, which preserves rotations but modifies boosts, the boost generators are deformed as follows:

$$K^i \equiv L_0^i + l_p p^i D \Rightarrow K^i = U^{-1} L_0^i U, \tag{3}$$

where l_p is the Planck length and L_0^i are the conventional generators of the Lorentz group, $L_{\mu\nu} = p_\mu(\partial/\partial p^\nu) - p_\nu(\partial/\partial p^\mu)$ [24]. U is a non-linear momentum map. U in momentum space becomes:

$$U_\mu(E, p_i) = (U_0, U_i) = (Ef_1, p_i f_2)\,. \tag{4}$$

By demanding plane-wave solutions to free field theories, $p_\mu x^\mu = p_0 x^0 + p_i x^i$, the momentum map in position space is given by:

$$U^\alpha(x) = \left(U^0, U^i\right) = \left(\frac{t}{f_1}, \frac{x^i}{f_2}\right), \tag{5}$$

which leads to the position space invariant (and hence the metric):

$$s^2 = \eta_{\alpha\beta} U^\alpha(x) U^\beta(x) = -\frac{t^2}{f_1^2} + \frac{(x^i)^2}{f_2^2}$$
$$\Rightarrow g_{\alpha\beta}(E) = \mathrm{diag}\left(-f_1^{-2}, f_2^{-2}, f_2^{-2}, f_2^{-2}\right), \tag{6}$$

where $\eta_{\alpha\beta}$ are the components of the Minkowski metric. In order to satisfy the correspondence principle, it is necessary to introduce a constraint on f_1 and f_2, namely

$$\lim_{E\to 0} f_k = 1,\ k = 1, 2 \quad \Rightarrow \quad \lim_{E\to 0} g_{\mu\nu}(E) = \eta_{\mu\nu}, \tag{7}$$

which restores Minkowski space in the low-energy limit [8]. In DSR, invariants of the modified Lorentz group are accompanied by a singularity in the momentum map U [24]. But in standard special relativity, the only energy invariant is the infinite one. Hence, to introduce a new invariant in the theory, the following relations must be fulfilled:

$$U(\tilde{E}) = \tilde{E} f_1(\tilde{E}) = \infty, \tag{8}$$

where $\tilde{E}$ is some new invariant energy scale. This constraint, however, is not used by all authors; phenomenologically motivated rainbow functions $f_{1,2}$ which do not fulfill the criterion (8) can be found in [10,1] among others.

The new metric $g_{\mu\nu}(E)$ defines a family of flat metrics parameterised by the energy E. Hence probe particles see "different universes"; they measure different cosmological quantities and travel on different geodesics, but share the same set of inertial frames [8].

In order to apply DSR to cosmology it is necessary to find the Friedmann–Lemaitre–Robertson–Walker (FLRW) metric, as modified by Rainbow Gravity. Here the following system of units is implied: $dx^0 = c_0 dt$, $c_0 = 1$, where c_0 is the low-energy limit of the energy-dependent speed of light, $c(E) \in [1, 0]$. Now, we need to modify the FLRW metric. The resulting expression is:

$$ds^2 = -\frac{dt^2}{f_1^2(E)} + \frac{a^2(t)}{f_2^2(E)} \gamma_{ij} dx^i dx^j, \tag{9}$$

where γ_{ij} represents the 3-metrics defined in Friedmann cosmology for the three different spacetime geometries ($K = 0, \pm 1$), and $a(t)$ is the scale factor. From the metric (9) we find the Einstein equations:

$$G_{\mu\nu}(E) = 8\pi G(E) T_{\mu\nu}(E) + g_{\mu\nu}(E)\Lambda(E), \tag{10}$$

where all quantities now vary with energy. The tensorial quantities gain their energy dependence from the rainbow functions contained in the metric, whereas $G(E)$ and $\Lambda(E)$ get theirs from renormalisation group flow arguments, as outlined in [8]. It is usually assumed that G and Λ have the same energy-dependence:

$$\begin{cases} G(E) = h^2(E) G_0 \\ \Lambda(E) = h^2(E)\Lambda_0 \end{cases} \tag{11}$$

where the index 0 indicates the standard table value. The function $h(E)$, which we will now call the 'scaling function' is constructed in such a way that the standard constants G_0, Λ_0 are recovered in the limit $E \to 0$. Such form of the h-dependence for the gravitational and cosmological constants allows the constancy of the vacuum energy density $\rho_\Lambda = \Lambda_0/8\pi G_0$.

3. Lorentz invariance violation in Rainbow Gravity

3.1. Lorentz invariance violation

Motivated by the notion of quantum foam coined by Wheeler [7], it has been suggested in theories of quantum gravity that Lorentz symmetry breaks down at high energies and short timescales [27,1]. A common approach when studying these effects from a phenomenological point of view is to assume an effective modified dispersion relation, manifesting itself at high energies [34,35,35]. In relation to that we consider a modified dispersion relation which for massless particles (whom we study from now on) takes the form:

$$p^2 = E^2 \ \rightarrow\ p^2 = E^2\left[1 + f(E)\right], \tag{12}$$

A modified dispersion relation such as the one in Eq. (12) would lead to highly energetic particles travelling slower or faster (depending on the quantum gravitational model) than their low-energy counterparts. For studies on Lorentz violation and possible observational tests, see [35–42,27].

In the framework of Lorentz Violation, it is often assumed that $f(E)$ in Eq. (12) can be expressed in a series expansion at low energies ($E \ll E_c$) [34,1,43]:

$$f(E) = \chi_1\left(\frac{E}{E_c}\right)^1 + \chi_2\left(\frac{E}{E_c}\right)^2 + \mathcal{O}\left[\left(\frac{E}{E_c}\right)^3\right], \tag{13}$$

where E_c is the energy scale at which Lorentz violating effects become strong, and the couplings $\chi_n = \pm 1$ ($n = 1, 2$) are determined by the dynamical framework being studied. It is also assumed that the effects of Lorentz violation enter in either a linear or a quadratic term, and thus the low-energy approximation of $f(E)$ can be written as [34]:

$$f(E) \approx \chi_n\left(\frac{E}{E_c}\right)^n. \tag{14}$$