Physics of the Dark Universe 18 (2017) 123–126

Contents lists available at ScienceDirect

Physics of the Dark Universe

journal homepage: www.elsevier.com/locate/dark

Gravitational wave bursts from Primordial Black Hole hyperbolic encounters

Juan García-Bellido [a,b,*], Savvas Nesseris [a]

[a] Instituto de Física Teórica UAM-CSIC, Universidad Autonóma de Madrid, Cantoblanco, 28049 Madrid, Spain
[b] CERN, Theoretical Physics Department, 1211 Geneva, Switzerland

ARTICLE INFO

Article history:
Received 17 September 2017
Received in revised form 4 October 2017
Accepted 14 October 2017

Keywords:
Gravitational waves
Primordial Black Holes
LIGO
LISA

ABSTRACT

We propose that Gravitational Wave (GW) bursts with millisecond durations can be explained by the GW emission from the hyperbolic encounters of Primordial Black Holes in dense clusters. These bursts are single events, with the bulk of the released energy happening during the closest approach, and emitted in frequencies within the AdvLIGO sensitivity range. We provide expressions for the shape of the GW emission in terms of the peak frequency and amplitude, and estimate the rates of these events for a variety of mass and velocity configurations. We study the regions of parameter space that will allow detection by both AdvLIGO and, in the future, LISA. We find for realistic configurations, with total mass $M \sim 60\,M_\odot$, relative velocities $v \sim 0.01\,c$, and impact parameters $b \sim 10^{-3}$ AU, for AdvLIGO an expected event rate is $\mathcal{O}(10)$ events/yr/Gpc^3 with millisecond durations. For LISA, the typical duration is in the range of minutes to hours and the event-rate is $\mathcal{O}(10^3)$ events/yr/Gpc^3 for both $10^3\,M_\odot$ IMBH and $10^6\,M_\odot$ SMBH encounters. We also study the distribution functions of eccentricities, peak frequencies and characteristic timescales that can be expected for a population of scattering PBH with a log-normal distribution in masses, different relative velocities and a flat prior on the impact parameter.

1. Introduction

Advanced LIGO has opened a new era of Gravitational Wave Astronomy, with the detection of at least three very massive Black Hole (BH) merger events [1–3], and probably a fourth one [4], in a few months of running. The signal corresponds to the inspiralling of two BH of several tens of solar masses in almost circular orbits, and the emission of GW leading to the final merger is in agreement, within experimental errors, with the predictions of General Relativity (GR). These massive BH binaries were rather unexpected, see however [5], suggesting a new population of very massive BH. This led to the speculation that AdvLIGO could have detected Primordial Black Holes (PBH) contributing to a significant fraction of Cold Dark Matter (CDM) [6–8], thus providing a natural explanation for its nature without resorting to exotic particles or modifications of gravity. Furthermore, these PBH could also provide the seeds for the Supermassive Black Holes (SMBH) found in the centers of the galaxies, as well as explaining the missing satellite and too-big-to-fail problems of CDM, thus solving several key problems in cosmology and galaxy formation in a unique and unified framework [9].

In the scenario of *clustered* PBH of Ref. [10], it is expected that a large fraction of BH encounters will not end up producing bounded systems, which would then inspiral, but rather produce a single scattering event, via a hyperbolic encounter. This could happen, e.g. if the relative velocity or relative distance of the two PBH is high enough that capture is not possible. The emission of GW in parabolic and hyperbolic encounters of compact bodies has been studied in the past in Refs. [11], and [12,13], respectively. These events generate *bursts* of gravitational waves, which can be sufficiently bright to be detected at distances up to several Gpc. For clustered PBH, the waveform and characteristic parameters of the GW emission in hyperbolic encounters are different to those of the inspiralling binaries, and both provide complementary information that can be used to determine the evolved mass distribution of PBH, as a function of redshift, as well as their spatial distribution.

Hyperbolic encounters are single scattering events where the majority of the energy is released near the point of closest approach, and have a characteristic peak frequency which is a function of only three variables, the impact parameter b, the eccentricity e and the total mass of the system M. Furthermore, the duration of such events is of the order of a few milliseconds to several hours, depending on those parameters. The case of inspiralling and merging PBH has been studied extensively, see e.g. Refs. [7,14], and estimated to produce a few tens of events/year/Gpc^3 in the range of $M_{\rm PBH} \sim \mathcal{O}(10-100)\,M_\odot$. In this letter we will show that,

* Corresponding author at: Instituto de Física Teórica UAM-CSIC, Universidad Autonóma de Madrid, Cantoblanco, 28049 Madrid, Spain.
E-mail addresses: juan.garciabellido@cern.ch (J. García-Bellido), savvas.nesseris@csic.es (S. Nesseris).

https://doi.org/10.1016/j.dark.2017.10.002
2212-6864/

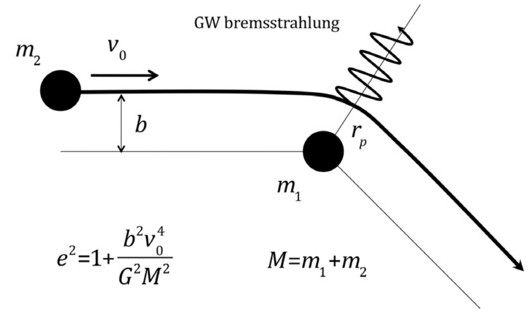

Fig. 1. The scattering of one BH of mass m_2 on another of mass m_1 induces the emission of gravitational waves which is maximal at the point of closest approach, r_p.

within the parameter space of the clustered PBH scenario [7,10], we expect a similar but somewhat lower rate of GW burst events in the millisecond range.

For a detector like AdvLIGO, such events will look like bursts with a characteristic frequency at peak strain amplitude. In fact, AdvLIGO has reported a few events of this type, which were attributed to accidental noise in the detectors [15]. However, events from hyperbolic encounters of PBH produce shapes that are rather similar to the "tear drop glitch" described in Ref. [16]. It is thus worth exploring the possibility that some of those events are actually PBH hyperbolic encounters. Their time–frequency profiles, discussed in this letter, could help the analysis of the AdvLIGO bursts and glitches. Moreover, if indeed these glitches arise from hyperbolic encounters of BH, they could be used to obtain valuable information about the PBH mass, velocity and spatial distribution.

2. Hyperbolic encounters of PBH

We thus consider a hyperbolic encounter between a BH of mass m_2 with asymptotic velocity v_0 against another BH of mass $m_1 = q\,m_2 \geq m_2$, see Fig. 1. The total mass of the system is then given by $M = (1+q)\,m_2$ and the reduced mass is $\mu = q\,M/(1+q)^2$. For an impact parameter b, the eccentricity of the hyperbolic orbit is given by $e \equiv \sqrt{1+b^2v_0^4/G^2M^2}$ [13], and the orbital trajectory is characterized in polar coordinates by $r(\varphi) = a(e^2-1)/(1+e\cos(\varphi-\varphi_0))$, where $b = a\sqrt{e^2-1}$, and $\varphi_0 = \arccos(-1/e)$. Then, the strain amplitude and power emitted in GW are given by

$$h_c = \frac{2G}{R\,c^4}\langle \ddot{Q}_{ij}\ddot{Q}^{ij}\rangle^{1/2}_{i,j=1,2} = \frac{2G\mu\, v_0^2}{R\,c^4}\, g(\varphi,\, e)\,, \tag{1}$$

$$\frac{dE}{dt} = -\frac{G}{45\,c^5}\langle \dddot{Q}_{ij}\dddot{Q}^{ij}\rangle = -\frac{32G\mu^2 v_0^6}{45\,c^5\,b^2}\, p(\varphi,\, e)\,, \tag{2}$$

where Q_{ij} is the reduced quadrupole moment of the BH encounter, and $p(\varphi,\, e)$ and $g(\varphi,\, e)$ are complicated bell-shaped functions of the angle φ, centered at φ_0. Here R corresponds to the distance from us, which in practice is the luminosity distance $d_L(z)$ of the event. It can then be shown that the maximum values of the power and strain amplitude only depend on the eccentricity of the orbit, $p_{\max}(e) = 9(e+1)^2/(e-1)^4$ and $g_{\max}(e) = 2\sqrt{18(e+1)+5e^2}/(e-1)$. The time dependence of these functions is given by, with $u = \varphi - \varphi_0$,

$$\frac{v_0\,t}{b} = \frac{e\sin u}{1+e\,\cos u} - \frac{2}{\sqrt{e^2-1}}\tanh^{-1}\left[\sqrt{\frac{e-1}{e+1}}\,\tan\frac{u}{2}\right] \tag{3}$$

from which we can estimate the characteristic time-scale of the event, $2\,t_{1/2}(e)$, corresponding to the full width at half maximum of the emission. In Fig. 2 we show the time dependence of the frequency shift and strain amplitude of GW in hyperbolic encounters, for the case $\beta = v_0/c = 0.1$, $b = 10^{-5}$AU and $M = 20\,M_\odot$. The colored regions correspond to different GW amplitudes. As can be seen, the shape of the emission in the time–frequency domain is exactly as expected for a GW burst, similar to those "tear-drop" bursts already observed in AdvLIGO [16].

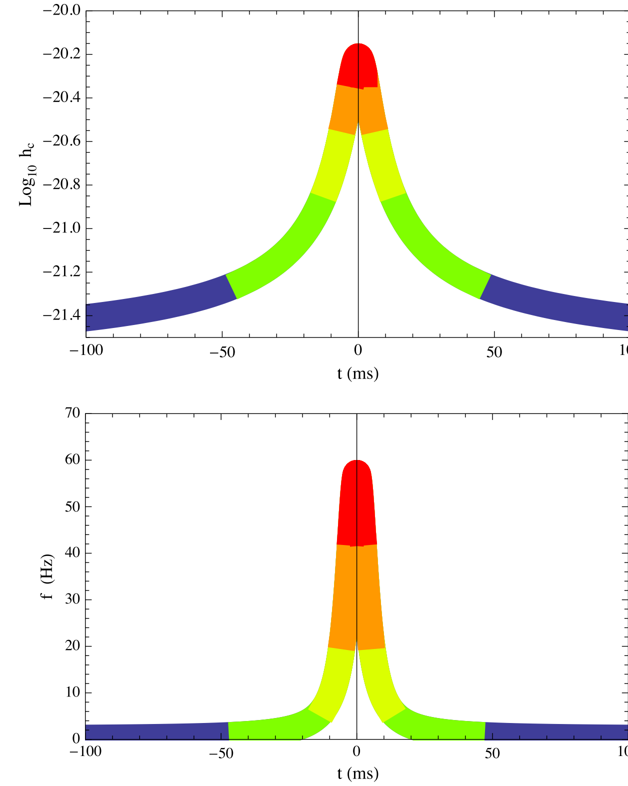

Fig. 2. The time dependence of the strain amplitude and frequency shift of GW in hyperbolic encounters, for the case $\beta = 0.1$, $b = 10^{-5}$AU and $M = 20\,M_\odot$. The different colors code different amplitudes, according to the top figure, and the peak frequency corresponds to maximum GW emission. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

It is useful to express the maximum strain amplitude (1) and power (2) in terms of physical quantities

$$h_c^{\max}(e) = 3.24\times 10^{-23}\,\frac{R_s(\mathrm{km})}{d_L(\mathrm{Gpc})}\,\frac{q\,\beta^2\,g_{\max}(e)}{(1+q)^2}\,, \tag{4}$$

$$P_{\max}(e) = 5.96\times 10^{26}\,\mathcal{L}_\odot\,\frac{q^2\,\beta^{10}}{(1+q)^4}\,\frac{(e+1)}{(e-1)^5}\,, \tag{5}$$

where the solar luminosity is $\mathcal{L}_\odot = 3.9\times 10^{33}$ erg/s, and $R_s = 2GM/c^2 = 3\,\mathrm{km}\,M/M_\odot$ is the Schwarzschild radius. The maximum frequency of the GW emission corresponds to $f_{\rm peak} = 0.32\,\mathrm{mHz}\,(e+1)/(e-1)\cdot\beta/b(\mathrm{AU})$, with the impact parameter in astronomical units. The product $f_{\rm peak}\cdot t_{1/2}(e)$ is a pure number that only depends on the eccentricity of the hyperbolic orbit. Since this can always be measured by the detector, we can estimate from here the parameter e and, substituting in (4) and (5), determine q and β for a given distance to the source. Note that the eccentricity provides a direct connection between the orbital parameters,

$$e^2 = 1 + \left(\frac{b}{10^{-5}\,\mathrm{AU}}\right)^2\left(\frac{\beta}{0.1}\right)^4\left(\frac{10\,M_\odot}{M}\right)^2. \tag{6}$$

For concreteness, we will now consider some typical examples in both the AdvLIGO and LISA range.

2.1. GW bursts in AdvLIGO

Let us consider first the hyperbolic encounter of two BH of 30 $M_\odot$ each, moving at a relative speed $\beta = 0.1$, with impact