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Interference of dark matter solitons and galactic offsets

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ABSTRACT

By performing numerical simulations, we discuss the collisional dynamics of stable solitary waves in the Schrödinger–Poisson equation. In the framework of a model in which part or all of dark matter is a Bose–Einstein condensate of ultralight axions, we show that these dynamics can naturally account for the relative displacement between dark and ordinary matter in the galactic cluster Abell 3827, whose recent observation is the first empirical evidence of dark matter interactions beyond gravity. The essential assumption is the existence of solitonic galactic cores in the kiloparsec scale. For this reason, we present simulations with a benchmark value of the axion mass $m_a = 2 \times 10^{-24}$eV, which is somewhat lower than the one preferred for cosmological structure formation if the field is all of dark matter ($m_a \approx 10^{-22}$ eV). We argue that future observations might bear out or falsify this coherent wave interpretation of dark matter offsets.

1. Introduction

The nature of dark matter is one of the most important open problems in fundamental physics. Projected experiments and astronomical observations are expected to shed new light on this question in the next decade [1].

In this context, the first evidence of dark matter (DM) non-gravitational self-interaction has been recently reported for the Abell 3827 cluster [2] ($z \approx 0.1$), where a displacement of the stars with respect to the maximum density of its DM halo has been observed, for some of the merging galaxies [3]. Possible explanations for this offset within the ΛCDM model comprise casual alignment with other massive structures that might influence the results from gravitational lensing, astrophysical effects affecting the baryonic matter, tidal forces or simply wrong identification of lensed images [4]. Even if these causes cannot be fully excluded, meticulous observations and simulations have shown that any such interpretation is unlikely to explain the collected data [4,5]. This tension with collisionless dark matter models [5] suggests the necessity of considering other possibilities as, *e.g.* self-interacting dark matter, that yields a drag force slowing down the galactic DM distribution while leaving the standard model sector unaffected [3–5]. Nonetheless, requiring that the drag induces the offset implies a lower bound for the cross section that is in tension with upper bounds derived from other observations, as carefully discussed in [6]. Thus, the Abell 3827 cluster presents a challenging puzzle that opens up questions of crucial importance to understand the nature and dynamics of DM.

In this work, we address the problem of the measured offset using the scalar field dark matter (ψDM) model [7–9], which considers a Bose–Einstein condensate (BEC) of non-relativistic ultralight axions (ULAs) of mass m_a subject to Newtonian gravity and that was introduced to solve difficulties of ΛCDM (*e.g.* missing satellites problem and cusp–core problem [10]), while maintaining the successful phenomenology of the model at cosmological scales [11,12]. Impressive numerical simulations [13] resolving largely different length scales have recently given support to this expectation. These extremely light scalar particles can arise in string theory constructions, *e.g.* [14] and other extensions of the standard model, *e.g.* [15]. Light scalars can also naturally appear as composites of hidden theories like the random UV field theory scenario [16].

We will show that the wave-like coherent nature of BECs severely affects the collisional dynamics of dark matter clumps, providing important effective forces even in the absence of explicit local interactions between the elementary dark matter constituents. We then discuss the possible relevance of this phenomenon to the puzzling observations described above.

2. Mathematical model

In the condensed scalar field scenario, the DM dynamics is governed by a Schrödinger–Poisson equation [17–20] for the

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mean-field wave-function ψ of the dark matter distribution:

$$i\hbar\partial_t\psi(t,\mathbf{x}) = -\frac{\hbar^2}{2m_a}\nabla^2\psi(t,\mathbf{x}) + \\ - G m_a^2\psi(t,\mathbf{x})\int\frac{|\psi(t,\mathbf{x}')|^2}{|\mathbf{x}'-\mathbf{x}|}d^3\mathbf{x}', \quad (1)$$

where $|\psi|^2$ is the particle number density, G the gravitational constant and t and $\mathbf{x}$ are time and position. For simplicity, we disregard cosmological evolution of the scale factor and the contribution of baryonic matter to the gravitational field, implicitly assuming that they do not play a prominent role in the processes studied below. Although a local interaction term $\lambda|\psi|^2\psi$ can be added to (1) [11,21,22], we will restrict ourselves to the simplest $\lambda = 0$ case [7,12,13] that, as we show below, is enough to describe the observed behavior. Notice, however, that drag forces appear in similar mathematical models for optical systems with non-linear terms $\lambda \neq 0$, *e.g.* [23].

Eq. (1) can be recast in terms of adimensional quantities:

$$i\,\partial_t\psi(t,\mathbf{x}) = -\frac{1}{2}\nabla^2\psi(t,\mathbf{x}) + \Phi(t,\mathbf{x})\psi(t,\mathbf{x}), \quad (2)$$

$$\nabla^2\Phi(t,\mathbf{x}) = 4\pi|\psi(t,\mathbf{x})|^2. \quad (3)$$

Following [24], the adimensional unit of length, time and mass correspond to:

$$\left(\frac{8\pi\,\hbar^2}{3m_a^2H_0^2\Omega_{m0}}\right)^{\frac{1}{4}} \approx 121\left(\frac{10^{-23}\text{ eV}}{m_a}\right)^{\frac{1}{2}}\text{ kpc}, \quad (4)$$

$$\left(\frac{3}{8\pi}H_0^2\Omega_{m0}\right)^{-\frac{1}{2}} \approx 75.5\text{ Gyr}, \quad (5)$$

$$\left(\frac{3}{8\pi}H_0^2\Omega_{m0}\right)^{\frac{1}{4}}\frac{\hbar^{\frac{3}{2}}}{m_a^{\frac{3}{2}}G} \approx 7\times10^7 M_\odot\left(\frac{10^{-23}\text{ eV}}{m_a}\right)^{\frac{3}{2}}. \quad (6)$$

We have taken $H_0 = 67.7$ km/(s Mpc) for Hubble's constant and $\Omega_{m0} = 0.31$ for the matter fraction of energy today.

Eq. (3) yields localized, radially symmetric, self-trapped robust solutions

$$\psi(t,\mathbf{x}) = \alpha e^{i\beta t}f(\sqrt{\alpha}|\mathbf{x}|), \qquad \Phi(t,\mathbf{x}) = \alpha\varphi(\sqrt{\alpha}|\mathbf{x}|), \quad (7)$$

which we will loosely call solitons. α is an arbitrary scaling constant, the propagation constant is $\beta = 2.454\alpha$, the soliton mass is $M_{sol} = \int|\psi|^2d^3\mathbf{x} = 3.883\sqrt{\alpha}$ and its diameter (full width at half maximum) is $d_{sol} = 1.380/\sqrt{\alpha}$. $f(\cdot)$ and $\varphi(\cdot)$ are functions that can be computed numerically. In terms of dimensionful quantities, the mass and size of the solitons are related by:

$$M_{sol}d_{sol} \approx \frac{5.36\,\hbar^2}{m_a^2G} \approx 4.6\times10^{10}\left(\frac{m_ac^2}{10^{-23}\text{ eV}}\right)^{-2}\text{ kpc}M_\odot, \quad (8)$$

where $M_\odot$ is the solar mass. In order to be reasonably self-contained, we provide a supplementary file (see appendix A) where we give more details on these solutions and also discuss the numerical methods used for the computations.

Finally, let us remark that these stationary states have been independently discussed in several physical contexts: foundations of quantum mechanics [20,25], cold trapped atoms [26,27], QCD-axions [28] and ultralight DM [29,30]. This often overlooked formal coincidence indicates that studies concerning Eq. (1) can have deeply multidisciplinary implications.

3. Numerical simulations

In ψDM, galactic dark matter distributions consist of a core which can be identified with a soliton surrounded by a background also governed by Eq. (3) and evolving in time and space with uncorrelated phases [13,24,31].

In this work, we propose that the offset of Abell 3827 [3] can come from the repulsion between coherent DM clumps (the solitonic cores) in phase opposition, without any extra local interactions. We show by numerical simulations that destructive interference can provide a large effective force acting on the cores. This repulsion between robust wave lumps is well known in soliton systems, from nonlinear optics [32,33] to atomic physics [34–36], where the mathematical description of the phenomena is similar to the theory of coherent DM waves.

In Ref. [3], observations of DM concentrations with mass of the order of $10^{11}M_\odot$ surrounding stellar distributions separated by around 10 kpc were presented. This value of $10^{11}M_\odot$ does not correspond to a galactic mass, but to clumps within the cluster that we will identify with solitonic cores. Most of the mass is in the halo, which behaves incoherently and therefore does not feel interferential forces. We will come back to this point in Section 4. Taking the aforementioned values for M_{sol} and d_{sol} in Eq. (8), we find $m_ac^2 \approx 2\times10^{-24}$ eV. We will fix this benchmark value for the simulations below. In Section 5, we provide a discussion on previous observational constraints on m_a and on their relevance to the phenomenon described here.

First, we have analyzed the collision of two DM solitons by numerically integrating (3) with the initial condition:

$$\psi(t=0,\mathbf{x}) = \alpha f\left(\sqrt{\alpha}|\mathbf{x}-\mathbf{x_0}|\right)e^{i(v\cdot x)} \\ + \alpha f\left(\sqrt{\alpha}|\mathbf{x}+\mathbf{x_0}|\right)e^{-i(v\cdot x-\Delta\phi)} \quad (9)$$

where $2|\mathbf{x_0}|$ is the initial separation, $2v$ the initial relative velocity, $\Delta\phi$ the relative phase and α is related to normalization as described in Section 2 (adimensional units). Previous studies of this sort with $\Delta\phi = 0$ can be found in [22,37]. We use a split-step pseudo-spectral algorithm, known as beam propagation method [38,39] (see the supplementary material for technical details). It is worth quoting other powerful numerical methods that have been recently developed for the Schrödinger equation with nonlocal terms [40,41].

As expected, see *e.g.* [42] for a discussion in nonlinear optics with a particular nonlinear potential, the outcome largely depends on the relative phase and speed. In the case of phase opposition, destructive interference creates a void region between the solitons which can induce a bounce. For phase coincidence, the solitons merge into a single matter lump (which for large initial velocities eventually splits again). Interference fringes appear for large velocities [22,37].

We must underline that in this work, for the first time to our knowledge, the effect of coherent DM waves on luminous matter has been calculated, by adding to our simulations test particles following classical trajectories in the gravitational field generated by the DM wave. These particles, initially located at the soliton centers, are a toy representation of the stars and can be shifted from the DM density peaks in a collision, as we show in Fig. 1 (see also supplementary videos 1 and 2). In Fig. 2, we plot the comparison between the trajectories of the point particle and the DM projected mass maximum. In order to check the limitations of this particle model, we have made use of the well known fact that Schrödinger equation can be cast into a hydrodynamic form through the Madelung transformation [43]. This allows us to develop a fluid toy model in which luminous matter is described as a spatially extended cloud (see supplementary material). As it can be seen in the inset of Fig. 2, both models display a good qualitative agreement.

Even if the collision in phase opposition is the simplest case, luminous vs. DM shifts can happen in more general situations. Fig. 3 and supplementary videos 3 and 4 show an example with