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Annihilation of relative equilibria in the gravitational field of irregular-shaped minor celestial bodies

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ABSTRACT

The rotational speeds of irregular-shaped minor celestial bodies can be changed by the YORP effect. This variation in speed can make the numbers, positions, stabilities, and topological cases of the minor body's relative equilibrium points vary. The numbers of relative equilibrium points can be reduced through the collision and annihilation of relative equilibrium points, or increase through the creation and separation of relative equilibrium points. Here we develop a classification system of multiple annihilation behaviors of the equilibrium points for irregular-shaped minor celestial bodies. Most minor bodies have five equilibrium points; there are twice the number of annihilations per equilibrium point when the number of equilibrium points is between one and five. We present the detailed annihilation classifications for equilibria of objects which have five equilibrium points. Additionally, the annihilation classification for the seven equilibria of Kleopatra-shaped objects and the nine equilibria of Bennu-shaped objects are also discussed. Equilibria of different objects fall into different annihilation classifications. By letting the rotational speeds vary, we studied the annihilations and creations of relative equilibria in the gravitational field of ten minor bodies, including eight asteroids, one satellite of a planet, and one cometary nucleus: the asteroids were 216 Kleopatra, 243 Ida, 951 Gaspra, 1620 Geographos, 2063 Bacchus, 2867 Steins, 6489 Golevka, and 101955 Bennu; the satellite of the planet was S16 Prometheus; and the comet was 1682 Q1/Halley. For the asteroid 101955 Bennu, which has the largest number of equilibria among the known asteroids, we find that the equilibrium points with different indices approach each other as the rotational speed varies and thus annihilate each other successively. The equilibrium in the gravitational field and the smooth surface equilibrium collides when the equilibrium touches the surface of the body.

1. Introduction

Recently, it has been found that the YORP effect (Yarkovsky–O'Keefe–Radzievskii–Paddack effect) is one of the most important influences on asteroid dynamics (Scheeres, 2007; Taylor et al., 2007; Lowry et al., 2014; Ševeček et al., 2015). YORP effect is caused by the effect of the electromagnetic radiation to the surface of the asteroid. The photons of the electromagnetic radiation from the Sun has momentum, the effect makes the variety of the angular momentum of the asteroid relative to the inertial space. Although the changes are very small, when the time is long enough, the changes of the angular momentum of the body may be very significant. An example is the asteroid 54509 YORP (2000 PH5). The YORP effect causes its rotational speed to increase, and the change in spin rate is $(2.0 \pm 0.2) \times 10^{-4}$ deg·day^{-2} (Taylor et al., 2007). Different asteroids have different YORP effects, the characteristic and the value of YORP effects depend on the size and irregular shape of the asteroids (Vokrouhlický; Čapek, 2002; Micheli and Paolicchi, 2008). Vokrouhlický and Čapek (2002) classified several types of YORP effects for asteroids; for instance, the YORP effect on Toutatis-shaped asteroids, including 433 Eros, 1998KY26, and 25143 Itokawa, belong to type I.

The YORP effect can lead to variations in the rotational speed, which causes the equilibrium shapes and equilibrium points of the asteroid to vary (Cotto-Figueroa et al., 2014; Hirabayashi and Scheeres, 2014). The relative equilibrium point is the point where the external force of the gravitational force and the Coriolis force equals zero. The gravitational force is caused by the irregular shape and mass distribution of the asteroid, while the Coriolis force is caused by the rotation of the asteroid. The changes in equilibrium shapes and equilibrium points influence the surface grain motion as well as the origin of asteroid pairs and triple asteroids (Walsh et al., 2012; Hirabayashi et al., 2015; Jiang et al.,

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2016b). To understand the characteristics of equilibrium shapes and equilibrium points, several important analytical and numerical results for conditions, stability, and classifications of equilibrium have been investigated.

Holsapple (2004) discussed the stability of equilibrium shapes by modeling the Solar System bodies as elastic-plastic solids. Richardson et al. (2005) investigated the shape and spin limits of rubble-pile Solar System bodies and found that the rubble-pile body consisting of a small number of particles is more stable than one consisting of a large number of particles. Sharma et al. (2009) produced the theoretical and numerical analysis of equilibrium shapes of gravitating rubble asteroids. Walsh et al. (2012) studied the disruption of asteroids and the formation of binary asteroids when the rubble-pile asteroids are spun up by the YORP effect. Hirabayashi and Scheeres (2014) calculated the surface shedding of the asteroid 216 Kleopatra and found that when the rotational speed changes from 5.385 h to 2.81 h, the outer equilibrium points will attach to the surface of the asteroid. This is the key influence on surface shedding and leads to the first-shedding condition, which happens because the radial acceleration is then oriented outwards (Scheeres et al., 2016).

To study the position, stability, and topological classification of equilibrium points, several authors have considered simple models to represent the stability and motion around equilibrium points. To understand the motion around equilibria near the elongated celestial bodies, Riaguas et al. (2001) investigated the nonlinear stability of the equilibria in the gravitational potential of a finite straight segment. Vasilkova (2005) used a triaxial ellipsoid to model the asteroid's shape and discussed the periodic motion around the equilibrium points of the triaxial ellipsoid. Palacián et al. (2006) calculated four equilibrium points in the potential of the finite straight segment and the invariant manifold near the equilibrium points. Guirao et al. (2011) discussed the position and stability of equilibria in a double-bar rotating system with changing parameters. However, a minor celestial body's irregular shape and gravitation are different from the simple models. The polyhedral model (Werner, 1994; Werner and Scheeres, 1996), which has sufficient faces to model the irregular shape and gravitation, is much more precise than the simple models. Jiang et al. (2014) presented the theory of local dynamics around equilibrium points and classified the non-degenerate equilibrium points into eight different cases. They used the polyhedral model and applied the theory to calculate the stability and topological cases around four different asteroids. Wang et al. (2014) used the classification presented in Jiang et al. (2014) and calculated the stability and topological classifications of equilibrium points around 23 minor celestial bodies; the shapes and gravitation models of these objects were also computed using the polyhedral model. Chanut et al. (2015) used the classification presented in Jiang et al. (2014) and found that there are one unstable equilibrium point and two linearly stable equilibrium points inside the asteroid 216 Kleopatra. Jiang et al. (2015) found that the equilibrium points in the potential of asteroid 216 Kleopatra may collide and annihilate each other as the rotational speed varies.

This paper investigates the annihilation and creation of equilibrium points of minor bodies when their rotation speeds vary. We take into consideration that different minor bodies may have different types of YORP effects, and different YORP effects lead to different changes in rotational speeds (Vokrouhlický; Čapek, 2002). We mainly want to calculate the annihilation classification, the kinds of bifurcations, and the annihilation positions of asteroids during changes in rotational speed. Therefore, we chose minor bodies with different YORP types, including 216 Kleopatra, 951 Gaspra, and 2063 Bacchus, etc. Asteroid 216 Kleopatra's YORP type is Type 2, 951 Gaspra's YORP type is Type 1, and 2063 Bacchus' YORP type is Type 4 (Micheli and Paolicchi, 2008). In addition, the dynamical behaviors of satellites of planets and cometary nuclei are different from those of asteroids, so we chose the satellite of planet S16 Prometheus and the comet 1682 Q1 Halley to analyze the annihilation and creation of equilibrium points around them when the rotational speeds vary.

The topological classifications and the indices of equilibrium points are analyzed. Then we present the annihilation classifications of relative equilibrium points as the parameters of the gravitational field vary. We discuss all the annihilation classifications for equilibria of objects that have five equilibrium points because most of the minor bodies have five equilibrium points. However, the Kleopatra-shaped objects may have seven equilibrium points as the rotational speed varies, and the Bennu-shaped objects may have nine equilibrium points as the rotational speed varies; therefore, we studied the annihilation classifications for seven equilibria of Kleopatra-shaped objects and nine equilibria of Bennu-shaped objects. We investigated ten objects, including the eight asteroids 216 Kleopatra, 243 Ida, 951 Gaspra, 1620 Geographos, 2063 Bacchus, 2867 Steins, 6489 Golevka, and 101955 Bennu, the satellite of planet S16 Prometheus, and the comet 1682 Q1/Halley. For asteroid 101955 Bennu, the results show that the equilibrium points with different indices approach each other when the rotational speed increases. The equilibrium points annihilate each other successively, and there are a total of four pairs of annihilations around asteroid 101955 Bennu.

The number of equilibrium points for almost axisymmetric bodies is quite variable, and elongated bodies can also have a different number of equilibrium points than those presented if there is some heterogeneity in the mass distribution. Although the number of equilibrium points for different shapes of celestial bodies seems to not have much value from the standpoint of mathematics, this phenomenological study has a wider significance. From this phenomenological study, one can summarize the characteristics of the equilibrium points, the kinds of bifurcations, as well as the annihilation positions, which are then helpful for a theoretical study.

2. Topological classifications and indices of equilibria

In a minor body's gravitational potential the gradient of the effective potential at the relative position $\mathbf{r} = (x, y, z)^{\mathrm{T}}$ can be written as Equation (1):

$$\begin{cases} \dfrac{\partial V(\mathbf{r})}{\partial x} = -\omega^2 x + \dfrac{\partial U(\mathbf{r})}{\partial x} \\ \dfrac{\partial V(\mathbf{r})}{\partial y} = -\omega^2 y + \dfrac{\partial U(\mathbf{r})}{\partial y}, \\ \dfrac{\partial V(\mathbf{r})}{\partial z} = \dfrac{\partial U(\mathbf{r})}{\partial z} \end{cases} \tag{1}$$

where $\mathbf{r}$ represents the relative position in the body-fixed frame, $\boldsymbol{\omega}$ represents the body's rotational angular velocity, ω represents the norm of $\boldsymbol{\omega}$, $U(\mathbf{r})$ represents the body's gravitational potential. The equilibrium point is defined as the zero point of the gradient of the effective potential (Jiang et al., 2014; Chanut et al., 2014). With its current rotation speed asteroid 1998 KY_{26} is found to have only one equilibrium point (Wang et al., 2014), 216 Kleopatra has seven equilibrium points (Hirabayashi and Scheeres, 2014; Wang et al., 2014; Chanut et al., 2015), 101955 Bennu has nine equilibrium points (Wang et al., 2014); other minor celestial bodies, including asteroids 4 Vesta, 243 Ida, 433 Eros, 951 Gaspra, 1620 Geographos, 1996 HW1, 2063 Bacchus, 2867 Steins, 4769 Castalia, 6489 Golevka, 25143 Itokawa and 52760 1998 ML_{14}, and satellites of planets J5 Amalthea, M1 Phobos, N8 Proteus, S9 Phoebe, and S16 Prometheus, as well as comets 1P/Halley, 9P/Tempel1, and 103P/Hartley2, all have five equilibrium points (Jiang et al., 2014; Wang et al., 2014). Most of these minor celestial bodies have one equilibrium point inside the body and the other equilibrium points outside except for 216 Kleopatra. 216 Kleopatra has three equilibrium points inside (Wang et al., 2014; Chanut et al., 2015) and four equilibrium points outside (Jiang et al., 2014).

The equation of motion relative to the equilibrium point (Jiang et al., 2014, 2015) can be expressed in the tangent space as Equation (2):