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Resonances in the asteroid and trans–Neptunian belts: A brief review

Tabaré Gallardo

Instituto de Física, Facultad de Ciencias, UdelaR, Iguá 4225, 11400 Montevideo, Uruguay

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ABSTRACT

Mean motion resonances play a fundamental role in the dynamics of the small bodies of the Solar System. The last decades of the 20th century gave us a detailed description of the dynamics as well as the process of capture of small bodies in coplanar or small inclination resonant orbits. More recently, semianalytical or numerical methods allowed us to explore the behavior of resonant motions for arbitrary inclination orbits. The emerging dynamics is very rich, including large orbital changes due to secular effects inside mean motion resonances. The process of capture in highly inclined or retrograde resonant orbits was addressed showing that the capture in retrograde resonances is more efficient than in direct ones. A new terminology appeared in order to characterize the properties of the resonances. Numerical explorations in the transneptunian region showed the relevance and the particular dynamics of the exterior resonances with Neptune which can account for some of the known high perihelion orbits in the scattered disk. Moreover, several asteroids evolving in resonance with planets other than Jupiter or Neptune were found and a large number of asteroids in three-body resonances were identified.

1. Introduction

An orbital resonance occurs when there is a commensurability between frequencies associated with the orbital motion of some bodies. These frequencies can include the mean motion n of the bodies (in which case we speak of a mean-motion resonance), or exclusively secular (low) frequencies associated with the long term evolution of the longitude of the nodes, Ω or the longitude of the perihelia, ϖ. In the dynamics of small Solar System bodies, these commensurabilities can generate two-body mean-motion resonances, involving the mean longitudes of the asteroid and one planet, three-body mean motion resonances, involving the mean longitudes of the asteroid and two planets, secular resonances involving longitudes of the perihelia and nodes and the Kozai-Lidov (KL) mechanism involving the asteroid's argument of the perihelion, $\omega = \varpi - \Omega$ (Shevchenko, 2017). A very concise but complete review on orbital resonances can be found in Malhotra (1998). In this paper we will refer only to two-body mean motion resonances (hereafter 2BRs) and three-body mean motion resonances (hereafter 3BRs) or, in general, mean motion resonances (hereafter MMRs). We will focus on the main advances of the 21st century, for earlier reviews the reader may consult for example Nesvorný et al. (2002), Malhotra (1998) or Peale (1976).

When an asteroid, or more generally, a minor body is in a 2BR with a planet of mass m_1 their mean motions verify $k_0 n_0 + k_1 n_1 \sim 0$ being n_0 and n_1 the mean motions of the minor body and the planet respectively and k_0 and k_1 small integers with different sign. In that case we say that the asteroid is in the resonance $|k_1| : |k_0|$. From theories developed and valid for low-inclination orbits it was proved that the resonance's strength is approximately proportional to $m_1 e^q$, being e the orbital eccentricity of the resonant minor body and where $q = |k_0 + k_1|$ is the order of the resonance (Murray and Dermott, 1999). It turns out that when considering low-inclination orbits, being $e < 1$, only low order resonances have dynamical interest (the high-order ones have negligible strength). The above criteria for resonant motion is just an approximation and the precise definition of the resonant state is given by the behavior of the critical angle $\sigma = k_0\lambda_0 + k_1\lambda_1 + \gamma$ being λ_i the quick varying mean longitudes and γ a slow evolving angle defined by a linear combination of the Ω_i and ϖ_i involved. A resonant motion is characterized by an oscillation, or *libration*, of the critical angle around a stable equilibrium point. In the low-inclination approximation they are located at $\sigma = 0^\circ$ or $\sigma = 180^\circ$ except for exterior resonances of the type 1:k and 1:1 resonances for which the locations depend on the orbital eccentricity, which is why they are known as asymmetric. A very special case of 2BR that has deserved a lot of attention along the history of celestial mechanics since Lagrange's times is the strong 1:1 resonance, that means coorbital objects like Jupiter's trojans and quasi-satellites.

On the other hand a minor body is in a 3BR with two planets of mass m_1 and m_2 when the mean motions verify $k_0 n_0 + k_1 n_1 + k_2 n_2 \sim 0$. From theories developed for zero inclination orbits it was proved that the resonance's strength is approximately proportional to $m_1 m_2 e^q$, where $q = |k_0 + k_1 + k_2|$ is the order of the resonance (Nesvorný and Morbidelli,

E-mail address: gallardo@fisica.edu.uy.

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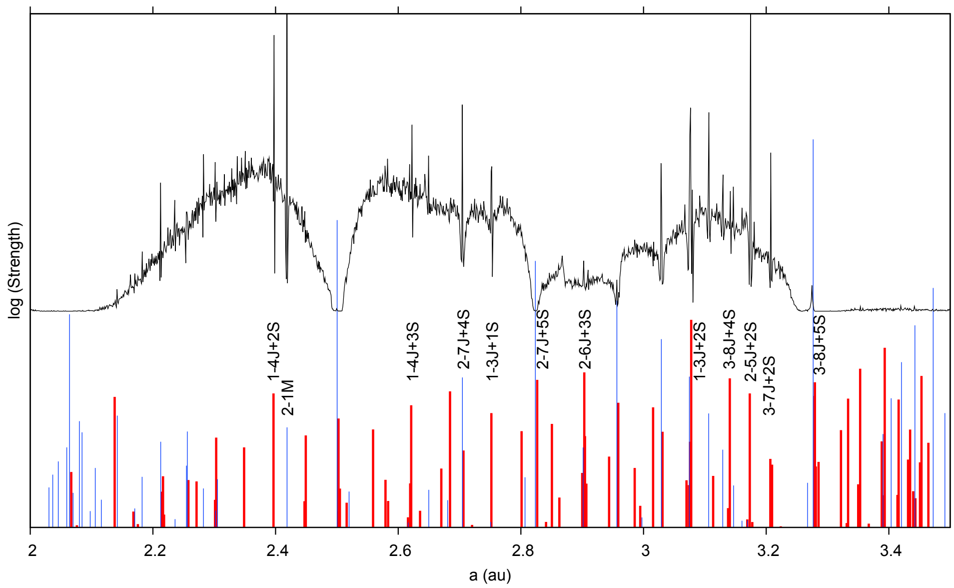

Fig. 1. Histogram of proper a (black line) taken from AstDyS (hamilton.dm.unipi.it/astdys) in a normalized scale plus 2BRs (thin blue lines) and 3BRs (thick red lines). The height associated to each resonance is in logarithmic scale and indicate the relative strength calculated for a test particle with $e = 0.2$, $i = 10°$ and $\omega = 60°$. The scales for 2BRs and 3BRs are different. Reproduced from Gallardo (2014). (For interpretation of the references to color in this figure legend, the reader is referred to the Web version of this article.)

1999). It is clear that being the masses expressed in units of solar masses the 3BRs are orders of magnitude weaker than 2BRs. It is important to stress that the 3BRs are not necessarily the result of the superposition of 2BRs between the intervening bodies as can be the case of the Galilean satellites of Jupiter or some extrasolar planetary systems (Gallardo et al., 2016). Three-body resonances exhibit also asymmetric equilibrium points as was showed by Gallardo (2014).

The commensurabilities above mentioned generate, in the long term, mean planetary perturbations on the minor body that are very different from the perturbations that a non resonant minor body experiences. The small planetary perturbation given at the right frequency gradually sums up instead of canceling out. Resonances do not emerge as instantaneous dynamical effects as, for example, a close encounter with a planet does. On the contrary, it is necessary to let the system evolve for several orbital revolutions in order that the minor body starts to feel the resonant gravitational potential.

The resonant motion is characterized by a regular small amplitude oscillation of the semi-major axis which preserves its mean value constant over time. This mean value is given by the corresponding mean motion n_0 defined by the resonant relation. These oscillations are correlated with oscillations in the orbital eccentricity and the librations of the critical angle σ. The frequency of the small amplitude oscillations are related to the resonance's strength: stronger resonances exhibit higher frequency oscillations (Ferraz-Mello, 2007). These oscillations are a protective mechanism that guarantees the constancy of the semimajor axis in front of other perturbations that the object can be exposed to. In particular, a chaotic diffusion of semi-major axis is immediately stopped (at least temporarily) if a capture in MMR occurs. This process is very

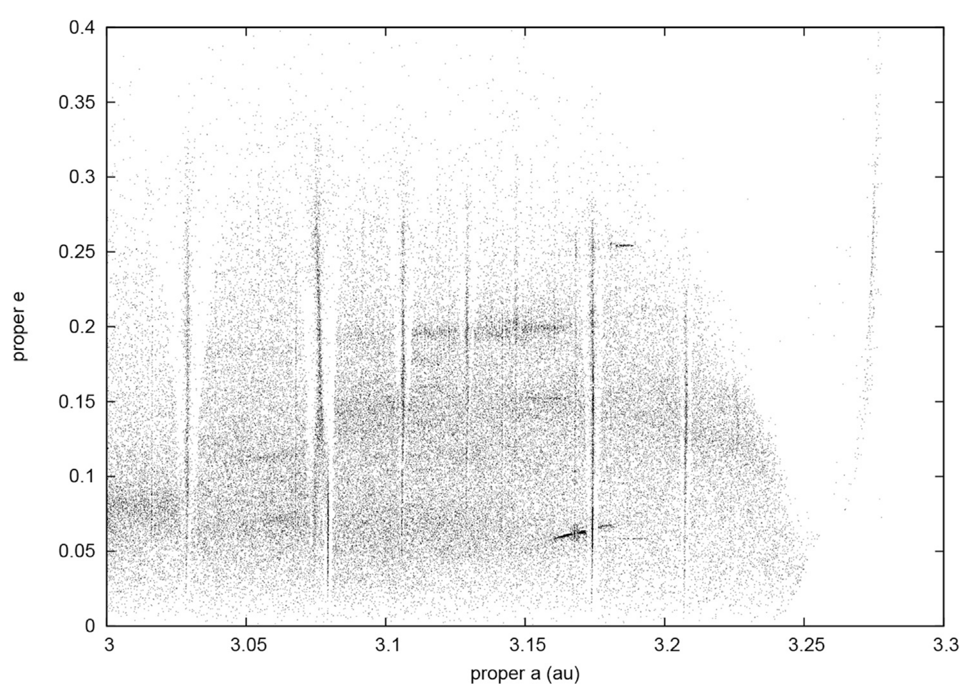

Fig. 2. Proper eccentricity versus proper semimajor axis taken from AstDyS. Resonances appear as vertical structures with increasing width for increasing e. The structure at $a \sim 3.075$ au is produced by a superposition of two resonances (see Fig. 8) and the border at the right is due to the 2:1 resonance with Jupiter.