Planetary and Space Science xxx (2017) 1–10

Contents lists available at ScienceDirect

Planetary and Space Science

journal homepage: www.elsevier.com/locate/pss

Lebedev acceleration and comparison of different photometric models in the inversion of lightcurves for asteroids

Xiao-Ping Lu [a,c,*], Xiang-Jie Huang [a], Wing-Huen Ip [a,b], Chi-Hao Hsia [a]

[a] Macau University of Science and Technology, Avenida Wai Long, Taipa, Macau, China
[b] Institute of Astronomy, National Central University, Taiwan
[c] Key Laboratory of Planetary Sciences, Chinese Academy of Sciences, Nanjing 210008, China

ARTICLE INFO

Keywords:
Cellinoid
Lightcurves
Asteroids
Lebedev
Photometric

ABSTRACT

In the lightcurve inversion process where asteroid's physical parameters such as rotational period, pole orientation and overall shape are searched, the numerical calculations of the synthetic photometric brightness based on different shape models are frequently implemented. Lebedev quadrature is an efficient method to numerically calculate the surface integral on the unit sphere. By transforming the surface integral on the Cellinoid shape model to that on the unit sphere, the lightcurve inversion process based on the Cellinoid shape model can be remarkably accelerated. Furthermore, Matlab codes of the lightcurve inversion process based on the Cellinoid shape model are available on Github for free downloading. The photometric models, i.e., the scattering laws, also play an important role in the lightcurve inversion process, although the shape variations of asteroids dominate the morphologies of the lightcurves. Derived from the radiative transfer theory, the Hapke model can describe the light reflectance behaviors from the viewpoint of physics, while there are also many empirical models in numerical applications. Numerical simulations are implemented for the comparison of the Hapke model with the other three numerical models, including the Lommel-Seeliger, Minnaert, and Kaasalainen models. The results show that the numerical models with simple function expressions can fit well with the synthetic lightcurves generated based on the Hapke model; this good fit implies that they can be adopted in the lightcurve inversion process for asteroids to improve the numerical efficiency and derive similar results to those of the Hapke model.

1. Introduction

As the primitive materials at the origin of our solar system, asteroids preserve information regarding planetary formation and their dynamical processes. Demeo and Carry (2014) show the path of solar system evolution in the perspectives of asteroidal composition and dynamical distribution. To more clearly investigate asteroids, including their surface compositions and inner structures, a few space missions were launched over the past decade. For example, Hayabusa 2, following its successful predecessor, was launched at the end of 2014 to visit the C-type asteroid (162173) Ryugu and will return a sample from the asteroid (Tsuda et al., 2013). Recently, in September of 2016, OSIRIX-REx was launched by NASA to visit the B-type asteroid (101955) Bennu (Lauretta et al., 2017). The asteroids will draw increasing attention, especially in the middle of 2018, when both of Hayabusa 2 and OSIRIX-REx will arrive at their respective target asteroids. Compared to the *in-situ* explorations by space missions, there are also more surveying observations from both ground-based and space-based telescopes. For example, NEO (Near Earth Objects) surveys, such as LINEAR, Pan-STARRS and so on, have collected vast numbers of photometric lightcurves (Jedicke et al., 2015). Moreover, the Sloan Digital Sky Survey (SDSS) and Wide-field Infrared Survey Explorer (WISE) (Wright et al., 2010) have been used to study asteroids via comprehensive measurements, including the colors and albedos (Michel et al., 2015). In addition, the Gaia satellite, launched at the end of 2013 by ESA, is implementing its 5-year regular observation mission for collecting the accurate positions and photometric sparse data of sources in the solar system, and its data release 1 is now formally published (Gaia Collaboration et al., 2016). Tanga et al. (2016) presented an overview of the asteroid observations by Gaia, covering the data processing and orbital inversion. Cellino and Dell'Oro (2012) indicated that the Gaia observations can be used to derive asteroid physical properties, including masses, sizes, average densities, spin properties, albedos, and reflectance spectra.

* Corresponding author. Macau University of Science and Technology, Avenida Wai Long, Taipa, Macau, China.
E-mail address: xplu@must.edu.mo (X.-P. Lu).

https://doi.org/10.1016/j.pss.2017.12.001
Received 25 July 2017; Received in revised form 29 November 2017; Accepted 1 December 2017
Available online xxxx

Based on the different shape models, the rotational periods and pole orientations of asteroids can be derived from the photometric observations, as well as their overall shapes. Generally, there are three commonly used shape models. The traditional triaxial ellipsoid shape model with three semi-axes is frequently applied in simulating the asteroids for searching their physical properties (Surdej and Surdej, 1978; Drummond et al., 2010; Lu et al., 2013). Furthermore, Muinonen et al. (2015) presented the method of asteroid lightcurve inversion based on application of the Lommel-Seeliger scattering law to the ellipsoid shape, and Cellino et al. (2015) applied this inversion method to the sparse photometric data. For more lightcurves observed in various viewing geometries, Kaasalainen et al. presented an inversion method based on the convex shape models (Kaasalainen et al., 1992; Kaasalainen and Lamberg, 1992). They represent the mapping function from the surface of the convex shape onto a unit sphere by the spherical harmonics, following the Minkowski process to determine the unique shape result (Kaasalainen and Torppa, 2001; Kaasalainen et al., 2001; Lamberg and Kaasalainen, 2001). Moreover, shape models for hundreds of asteroids obtained by this inversion method are available in the DAMIT database (Ďurech et al., 2010). To consider an intermediate shape between the ellipsoid and the convex shape, the Cellinoid shape model, which accounts for asymmetric shape features, was first presented by Cellino et al. (1989) to simulate asteroids. Lu and Ip (2015) completed the whole lightcurve inversion process based on this intermediate shape and first called it 'Cellinoid'. Furthermore, Lu et al. (2016) applied the Cellinoid shape model to the Hipparcos data set and confirmed that it can perform particularly well in the case of sparse photometric data, such as the Hipparcos data and the future analogous Gaia data set.

In the lightcurve inversion process the numerical routine of simulating the photometric brightness over the specified shape model consumes the most CPU time. Lebedev quadrature is an efficient method to numerically calculate the surface integral on the unit sphere (Lebedev and Laikov, 1999). Kaasalainen et al. (2012) introduced the optimal computation of brightness integrals by adopting the Lebedev quadrature in their convex inversion. Lu et al. (2013) also attempted to apply the Lebedev quadrature to the lightcurve inversion process based on ellipsoid shape model and largely accelerated the algorithm. Before successfully inducing the analytical formula of the brightness integral for the Cellinoid shape model, it should be very useful to apply the Lebedev quadrature to the lightcurve inversion process for reducing computational cost. In this article the mapping from the surface of Cellinoid shape to the surface of unit sphere is presented and the brightness simulation can be accelerated substantially by applying the Lebedev quadrature.

The scattering laws, which describe the light reflectance behaviors, can be employed in the lightcurve inversion process of asteroids based on photometric observations. Hapke (2012) described the theory of reflectance in details and introduced the Hapke model to illustrate the bidirectional reflectance of planetary photometry, incorporating the opposition effect, regolith porosity and surface roughness based on the single-particle scattering theory. Based on radiative transfer theory, the Hapke model can describe the physical properties of a planetary surface; however, its complex formula expression is not convenient for use in numerical simulation, especially in lightcurves inversion. There are many other photometric models, such as Lommel-Seeliger (Hapke, 2012) and Minnaert (1941), as well as the scattering function adopted in Kaasalainen's inversion method (Kaasalainen et al., 2001). These models are numerically easy to implement in the lightcurve inversion process. Takir et al. (2015) compared the different photometric models, including the Minnaert and Lommel-Seeliger models, in simulating the ground-based photometric phase curve data of the OSIRIS-REx target asteroid (101955) Bennu. Karttunen and Bowell (1989) concluded that the variations of lightcurves depend very strongly on the body shape by analyzing synthetic lightcurves and phase curves, generated from various asteroid models using the Lumme-Bowell scattering law. Therefore, in this article, three different photometric models, namely, the Lommel-Seeliger, Minnaert, and Kaasalainen models, are compared in simulating the synthetic lightcurves generated from the Hapke photometric model and based on various shape models. An appropriate photometric model is expected to be found for use in the lightcurve inversion process that balances efficiency and accuracy.

In Section 2, Lebedev quadrature will be introduced in details, as well as the corresponding brightness integral on the unit sphere, based on mapping from the Cellinoid shape model. Next, the four photometric models of Hapke, Lommel-Seeliger, Minnaert and Kaasalainen will be presented in Section 3. Subsequently, the numerical simulations comparing different photometric models will be presented. Section 4 will discuss the implications of the numerical results. The conclusions and plans for future works will conclude the article in Section 5.

2. Lebedev acceleration

2.1. Lebedev quadrature

Lebedev and Laikov (1999) presented an efficient tool of the surface integral on the unit sphere that is often applied in the numerical calculation of the surface integral in the spherical coordinate system. Lebedev quadrature can approximately transform the surface integral of the function f over the unit sphere S,

$$I = \iint f(\Omega)\, d\Omega = \int_0^{\pi} \sin(\theta)\, d\theta \int_0^{2\pi} d\varphi f(\theta, \varphi), \tag{1}$$

to a linear combination of the weights w_i and the function values $f(\theta_i, \varphi_i)$ at the Lebedev grids with the grid size, i.e., Lebedev degree N,

$$I \approx \sum_{i=1}^{N} w_i f(\theta_i, \varphi_i), \tag{2}$$

where the sum of the weight w_i is equal to the surface area W of the unit sphere,

$$W = \sum_{i=1}^{N} w_i = 4\pi.$$

Compared with the two-dimensional discretization of the surface integral (1) on the unit sphere, the linearly sum (2) makes the calculation more efficient, and fewer Lebedev grid points are required to obtain similar accuracy to the commonly used triangularization scheme. Fig. 1 shows the discretized unit sphere with Lebedev grids in different degrees N and their corresponding volumes. As the benchmark, the volume of the unit sphere can be derived analytically, $V = 4\pi/3 \approx 4.1888$. As the Lebedev degree increases, the discretized sphere approaches the unit sphere. In particular, with the degree of only $N = 302$, the volume of discretized unit sphere is 4.1109, with the deviation of 1.86% to the benchmark value of 4.1888. Fig. 2 shows a comparison between the Lebedev discretization and the traditional triangularization with nearly equal areas.

2.2. Brightness integral

As described by Lu and Ip (2015), the brightness integral based on the Cellinoid shape model can be expressed as,

$$B(E_0, E) = \iint_{C+} S(\mu, \mu_0, \alpha)\, ds, \tag{3}$$

where $S(\mu, \mu_0, \alpha)$ is the scattering function with the definitions of μ and μ_0, representing the projections of viewing and illuminating unit vectors on the unit normal vector of each facet ds, α is the solar phase angle, and C_+ is the part of the Cellinoid surface that is both illuminated by Sun and observable from Earth, i.e., $\mu_0 > 0, \mu > 0$.

Following the definition of the brightness integral (3), the numerical quadrature can be applied to calculate it, as the analytical formula has