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Orbit determination based on meteor observations using numerical integration of equations of motion

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ABSTRACT

Recently, there has been a worldwide proliferation of instruments and networks dedicated to observing meteors, including airborne and future space-based monitoring systems . There has been a corresponding rapid rise in high quality data accumulating annually. In this paper, we present a method embodied in the open-source software program "Meteor Toolkit", which can effectively and accurately process these data in an automated mode and discover the pre-impact orbit and possibly the origin or parent body of a meteoroid or asteroid. The required input parameters are the topocentric pre-atmospheric velocity vector and the coordinates of the atmospheric entry point of the meteoroid, i.e. the beginning point of visual path of a meteor, in an Earth centered-Earth fixed coordinate system, the International Terrestrial Reference Frame (ITRF). Our method is based on strict coordinate transformation from the ITRF to an inertial reference frame and on numerical integration of the equations of motion for a perturbed two-body problem. Basic accelerations perturbing a meteoroid's orbit and their influence on the orbital elements are also studied and demonstrated. Our method is then compared with several published studies that utilized variations of a traditional analytical technique, the zenith attraction method, which corrects for the direction of the meteor's trajectory and its apparent velocity due to Earth's gravity. We then demonstrate the proposed technique on new observational data obtained from the Finnish Fireball Network (FFN) as well as on simulated data. In addition, we propose a method of analysis of error propagation, based on general rule of covariance transformation.

1. Introduction

Improving existing techniques, derived from ground based observations, and developing new methods that more accurately determine meteor orbits are among the goals of meteor astronomy.

Typically, an analysis of meteor observations yields the azimuth and the inclination of the atmospheric trajectory (i.e. a topocentric radiant) of a meteor, its apparent velocity, and the coordinates of the origin of its visual path. These data are sufficient to determine the pre-impact heliocentric orbit of the meteoroid. Knowing the heliocentric orbit may lead to discovery of the meteoroid's parent body or its origin. In the past several authors conducted an analysis of the dynamical evolution of asteroid or meteoroid's orbits. To implement this, they employed a numerical integration of equations of motions over a long time backwards from the impact date. For example, in the works (de la Fuente Marco and de la Fuente Marcos, 2013; Trigo-Rodriguez et al., 2015) a backward integration was performed for period of at least 10000 years. If there are sufficient uncertainties in the initial conditions, then they can negate the value of this type of analysis.

It is obvious, that the greatest changes to an asteroid's or meteoroid's orbit occur just prior to its encounter with Earth. Thus, in determining the orbit we have to be very precise and careful when we consider the influences of all the corrections and perturbing forces. As a meteoroid approaches Earth, its orbit changes, primarily under the influence of Earth's gravity. Currently, a “zenith attraction” technique is widely used to account for this effect. The zenith attraction method employs corrections to compensate for Earth's gravity effect on the direction of the meteor's trajectory and its apparent velocity.

This technique was described and used to determine the orbits of meteors registered by the cameras of the European Fireball Network in Ceplecha (1987). Implementations of the zenith attraction method have also been introduced in several software packages, for example, in Zoladek (2011) and in Langbroek (2004). In recent analysis by Clark and Wiegert (2011) and Zuluaga et al.

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(2013) a backward numerical integration was performed in place of the traditional calculation of zenith attraction corrections. The authors took into account the perturbations from Earth and other planets as point masses.

In addition to the influence of Earth, as a point mass, a meteoroid's orbit is also influenced by the attraction of the Moon, atmospheric drag, the non-central part of the Earth's gravity, and by the attraction of other Solar system planets. In the past some of these effects could be neglected due to the low accuracy determination of the apparent track of a meteor. However, the recent, more precise, data collected by the dedicated fireball networks (which are well established in Central Europe, USA, Canada, Finland, Spain, Australia and other countries), do allow for an accurate determination of a meteor's trajectory. The orbital parameters of several meteorite producing fireballs, as detected by the instruments of these fireball networks, were precisely derived. See e.g., the summary table in Jenniskens et al. (2012) and in Trigo-Rodriguez et al. (2015). Furthermore, the need for precise data reduction methods is justified by more elaborate techniques proposed today for meteor observations, such as international airborne campaigns (Vaubaillon et al., 2015) and possible future space-based monitoring systems (Bouquet et al., 2014).

2. Traditional method

Corrections for the Earth's gravitational influence upon a meteor's direction were first proposed by Schiaparelli during the second half of nineteenth century. And since its introduction it has been widely used. Detailed analysis of this technique was performed in Andreev (1990) and, in a numerical simulation, by Gural (2001). First of all, the diurnal aberration is taken into account. The influence of gravity on the zenith distance and velocity of the meteoroid is described as follows:

$$Z = Z` + \Delta Z, \tag{1}$$

$$\Delta Z = 2\arctan\left[\frac{V - V_g}{V + V_g} tg\left(\frac{Z`}{2}\right)\right], \tag{2}$$

$$V_g^{\,2} = V^2 - \frac{2GM_\oplus}{R_\oplus + h}. \tag{3}$$

where Z is the true zenith distance, Z' is the apparent zenith distance, V is the apparent velocity of meteoroid, V_g is geocentric velocity of meteoroid, $GM_\oplus$ is geocentric gravitational constant, $R_\oplus$ is the Earth's mean radius, h is a beginning height of a meteor. The zenith attraction method applies these corrections for zenith distance and velocity.

Next, the velocity components are transformed to the inertial coordinate system. Finally, the position and velocity of the Earth, relative to the Sun, is calculated. Earth's position is taken as the position of the meteoroid, corrected to its atmospheric intercept point, in a heliocentric coordinate system, and the components of the meteoroid's velocity are added to the components of the Earth's velocity. The calculated heliocentric state vector of the meteoroid may be then transformed into the orbital elements. As will be shown below, the traditional technique works well for fast meteoroids, but may give errors for low-velocity meteors. As calculated from Eqs. (1)–(3), both the velocity and the zenith distance of the meteoroid are continuously changing as the meteoroid approaches Earth.

3. Description of the proposed method

In contrast to the zenith attraction method, as discussed above, our study employs strict transformations of coordinate systems and velocity vectors recommended by the IAU International Earth Rotation and Reference Systems Service in the IERS Conventions (2010), see Petit and Luzum (2010), SOFA (2013) and a backward numerical integration (Plakhov et al., 1989) of the equations of motion.

As a first step, we transform the velocity components from the topocentric horizontal coordinate system to the Greenwich equatorial coordinate system

$$\begin{pmatrix} Vx \\ Vy \\ Vz \end{pmatrix} = \mathbf{M}^{\mathrm{T}} \begin{pmatrix} Vn \\ Ve \\ Vu \end{pmatrix}, \tag{4}$$

$$\mathbf{M} = \mathbf{Q_1}\mathbf{R_2}(\phi - 90^\circ)\mathbf{R_3}(\lambda), \tag{5}$$

where Vn, Ve, Vu and Vx, Vy, Vz are components of the velocity of a meteor in the topocentric horizontal and in the Greenwich equatorial coordinate systems, respectively, matrix $\mathbf{M}^{\mathrm{T}}$ is rotation matrix from the topocentric horizontal to the Greenwich equatorial coordinate systems. $\mathbf{R_2}$, $\mathbf{R_3}$ and $\mathbf{Q_1}$ are appropriate rotation matrices and a mirror matrix, respectively, and φ and λ are the geodetic latitude and longitude of the initial point of the meteor, respectively.

Next, the diurnal aberration is taken into account as

$$\begin{pmatrix} \Delta Vx \\ \Delta Vy \\ \Delta Vz \end{pmatrix} = -\,\omega_e \begin{pmatrix} (N + h)\cos\phi\,\sin\lambda \\ (N + h)\cos\phi\,\cos\lambda \\ 0 \end{pmatrix}, \tag{6}$$

where h is geodetic height, N is radius of curvature of Earth ellipsoid prime vertical

$$N = R_e / \sqrt{1 - e^2 \sin^2\varphi}, \tag{7}$$

where $R_\oplus$ is the equatorial radius of the Earth and $\omega_\oplus$ is the angular rotation velocity of the Earth.

Therefore, apparent geocentric velocity components Vx_{geo}, Vy_{geo}, Vz_{geo} are

$$\begin{pmatrix} Vx_{geo} \\ Vy_{geo} \\ Vz_{geo} \end{pmatrix} = \begin{pmatrix} Vx \\ Vy \\ Vz \end{pmatrix} + \begin{pmatrix} \Delta Vx \\ \Delta Vy \\ \Delta Vz \end{pmatrix} \tag{8}$$

The transformation of the geocentric radius vector of the meteor's entry point and contributed components of the Earth's velocity from an Earth fixed, geocentric coordinate system ITRF2000, to a Geocentric Celestial Reference System (GCRS), version ICRF2 (J2000), are conducted according to the IERS Conventions (2010). The general formulas describing this transformation are given below. For the velocity vector

$$\begin{pmatrix} Vx_{in} \\ Vy_{in} \\ Vz_{in} \end{pmatrix} = \mathbf{R}^{\mathrm{T}} \begin{pmatrix} Vx_{geo} \\ Vy_{geo} \\ Vz_{geo} \end{pmatrix}, \tag{9}$$

and for the geocentric radius vector of entry point, which is also the starting point of further integration

$$\begin{pmatrix} X_{in} \\ Y_{in} \\ Z_{in} \end{pmatrix} = \mathbf{R}^{\mathrm{T}} \begin{pmatrix} X_{geo} \\ Y_{geo} \\ Z_{geo} \end{pmatrix}. \tag{10}$$