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Phobos: Observed bulk properties

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ABSTRACT

This work is a review of the mass determinations of the Mars moon Phobos by spacecraft close flybys, by solving for the Martian gravity field and by the analysis of secular orbit perturbations. The absolute value and accuracy is sensitive on the knowledge and accuracy of the Phobos ephemeris, of the spacecraft orbit, other perturbing forces acting on the spacecraft and the resolution of the Martian gravity field besides the measurement accuracy of the radio tracking data. The mass value and its error improved from spacecraft mission to mission or from the modern analysis of "old" tracking data but these solutions depend on the accuracy of the ephemeris at the time of observation. The mass value seems to settle within the range of $GM_{Ph}=(7.11\pm0.09)\times10^{-4}\ \mathrm{km^3\,s^{-2}}$ which covers almost all mass values from close flybys and "distant" encounters within its $3-\sigma$ error (1.5%). Using the volume value determined from MEX HRSC imaging, the bulk density is $(1873\pm31)\ \mathrm{kg\,m^{-3}}$ ($3-\sigma$ error or 1.7%), a low value which suggests that Phobos is either highly porous, is composed partially of light material or both. The determination of the gravity coefficients C_{20} and C_{22} from the Mars Express 2010 close flyby does not allow to draw conclusion on the internal structure. The large errors do not distinguish whether Phobos is homogeneous or not. In view of theories of the Phobos′ origin, one possibility is that Phobos is not a captured asteroid but accreted from a debris disk in Mars orbit as a second generation solar system object.

1. Introduction

Both Mars moons, Phobos and Deimos, were discovered and named by Asaph Hall (Hall, 1878) at the Naval Astronomical Observatory in 1877 as a result of a systematic search during the opposition of Mars. There have been speculations on the origin, nature, formation and evolution of the Mars moons: asteroid capture by Mars (Burns, 1992), simultaneous formation with Mars, formation in orbit from a debris disk of a previously larger body destroyed by gravitational gradient forces near Mars (Singer, 2007) and re-accretion of impact debris blasted into Mars orbit (Craddock, 2011) are prominent among these. Rosenblatt (2011) gives an overview on these formation scenarios.

Important for the understanding and investigation of formation scenarios, origin, nature and internal structure is the knowledge of the bulk parameters mass, density, gravity field, shape and porosity which will constrain any model of Phobos. These bulk parameters are also important for the navigation of a spacecraft near Phobos and eventual its landing and must be known at a certain precision.

2. Mass determination of small bodies

2.1. Mass and gravity field

The gravity potential of a body with bulk density ρ, axes a, b, c, reference radius R_0 and total mass M is usually expressed as Vallado (2001)

$$\Phi(r,\theta,\varphi)=\frac{GM}{r}\left\{1+\sum_{l=2}^{\infty}\sum_{m=0}^{l}\left(\frac{R_0}{r}\right)^{l}P_{lm}(\cos\theta)[C_{lm}\cos(m\varphi)+S_{lm}\sin(m\varphi)]\right\} \tag{1}$$

where r,θ,ϕ are the spherical coordinates of a test particle in distance r to the center of mass of the body of mass M; $G=6.6738\times10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$ is the gravitational constant, $P_{lm}(cos\theta)$ are the associated Legendre polynomials; the C_{lm} and S_{lm} are the expansion coefficients of degree l and order m. These expansion coefficients are defined as:

$$\left.\begin{matrix}C_{lm}\\ S_{lm}\end{matrix}\right\}=-\frac{(2-\delta_{m0})(l-m)!}{MR_0^l\ \ (l+m)!}\int_0^{R_0}\int_{-1}^{1}\int_0^{2\pi}\rho(r,\theta,\varphi)\times P_{lm}(\cos\theta)r^{l+2}\left\{\begin{matrix}\cos(m\varphi)\\ \sin(m\varphi)\end{matrix}\right\}dr\ d\cos\theta d\varphi \tag{2}$$

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and contain explicitly the internal mass or density distribution $\rho(r, \theta, \phi)$. Truncated after degree and order two, the gravity potential reads as

$$\Phi(r,\theta,\varphi) = \frac{GM}{r} + \frac{GM}{r}\left(\frac{R_0}{r}\right)^2 C_{20}P_{20}(\cos\,\theta) + \frac{GM}{r}\left(\frac{R_0}{r}\right)^2 C_{22}P_{22}(\cos\,\theta)\cos(2\varphi) \tag{3}$$

with

$$P_{20}(\cos\,\theta) = \tfrac{3}{2}\cos^2\theta - \tfrac{1}{2}$$

$$P_{22}(\cos\,\theta) = 3\,\sin^2\theta$$

$$C_{20} = -\frac{1}{MR_0^2}\int_0^{R_0}\int_{-1}^{1}\int_0^{2\pi}\rho(r,\theta,\varphi)P_{20}(\cos\,\theta)r^4 dr\;d\,\cos\,\theta\,d\varphi \tag{4}$$

$$C_{22} = -\frac{1}{12MR_0^2}\int_0^{R_0}\int_{-1}^{1}\int_0^{2\pi}\rho(r,\theta,\varphi)P_{22}(\cos\,\theta)r^4 \times \cos(2\varphi)\,dr\;d\,\cos\,\theta\,d\varphi \tag{5}$$

The first term in (3) is identified as the gravity potential of a point mass M. The second and third term in (3) describe extra contributions to the potential caused by an asymmetric shape and mass inhomogeneities along the axes of the principal moments of inertia.

Assuming a homogeneous mass distribution with constant density ρ, the C_{20} and C_{22} are represented by their axis dimensions of the ellipsoid with axis a, b, c embracing the shape of Phobos:

$$C_{20} = \frac{1}{5R_0^2}\left\{c^2 - \frac{1}{2}(a^2+b^2)\right\} \tag{6}$$

$$C_{22} = \frac{1}{20R_0^2}(b^2-a^2) \tag{7}$$

with the mean radius $R_0^2 = 1/3(a^2+b^2+c^2)$. Table 1a gives the values of the main axes of the Phobos ellipsoid, the mean radius R_0 and the derived C_{20} and C_{22} as a "function of time" represented by three selected publications (Bursa et al., 1990; Borderies and Yoder, 1990, Willner et al., 2010) demonstrating the improvement in the knowledge of the shape of Phobos. The modern values by Willner et al. (2010) yield $C_{20} = -0.106$ and $C_{22} = -0.015$ for a homogeneous Phobos.

Another approach to estimate C_{20} and C_{22} for a homogeneous body is by the moments of inertia

$$C_{20} = \frac{1}{MR_0^2}\left\{\tilde{C} - \frac{1}{2}(\tilde{A}+\tilde{B})\right\} \tag{8}$$

$$C_{22} = \frac{1}{4MR_0^2}(\tilde{B}-\tilde{A}) \tag{9}$$

with $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ as the principal moments of inertia, $\tilde{C} > \tilde{B} > \tilde{A}$. The $A = \tilde{A}/MR_0^2$, $B = \tilde{B}/MR_0^2$, $C = \tilde{C}/MR_0^2$ are called the normalized moments of inertia.

Table 1b lists the normalized principal moments of inertia from the same references as in Table 1a. Bursa et al. (1990) estimated the moments of inertia from a numerical dynamical model in preparation for the Soviet Phobos mission; Borderies and Yoder (1990) estimated the normalized moments of inertia from Duxbury and Callahan (1989) topographic model; Willner et al. (2010) estimated the moments of inertia from the shape model derived from MEX-HRSC imaging, also for a homogeneous Phobos. All values in Tables 1a and 1b represent an evolution over time on improved bulk parameters and shape. The normalized moments of inertia A, B, C do increase, because of the product $M_{Ph}R_0^2$ (Table 1b) which in fact decreased.

We adopt the value representative for a homogeneous Phobos from the shape model by Willner et al. (2010). The errors are computed from the errors of the semi axes of the embracing ellipsoid given by ± 250 m

$$C_{20,\mathrm{homogeneous}} = -0.106 \pm 0.01 \tag{10}$$

$$C_{22,\mathrm{homogeneous}} = -0.015 \pm 0.002 \tag{11}$$

Any deviation of the observed C_{20} and C_{22} values of the Phobos gravity field from those above for a homogeneous Phobos (Eqs. (10) and (11)) will indicate a deviation from a homogeneous mass distribution.

The convention in satellite geodesy is to use normalized associated Legendre polynominals $\overline{P}_{lm}$ and normalized gravity expansion coefficients $\overline{C}_{lm}, \overline{S}_{lm}$, where

$$\overline{C}_{lm} \times \overline{P}_{lm} = C_{lm} \times P_{lm}$$

The normalization is defined as (Vallado, 2001):

$$\overline{C}_{lm} = \Pi_{lm} \times C_{lm}$$

$$\overline{S}_{lm} = \Pi_{lm} \times S_{lm}$$

Table 1a
Ellipsoid dimensions of the figure of Phobos, mean radius and derived C_{20} and C_{22}.

Reference	*a* (m)	*b* (m)	*c* (m)	R_0 (m)	C_{20}	C_{22}
Bursa et al. (1990)	13,218	10,587	9,352	11,170	−0.090	−0.025
Borderies and Yoder (1990)	13,300	11,360	9,230	11,420	−0.104	−0.018
Willner et al. (2010)	13,000	11,390	9,070	11,270	−0.106	−0.015

Table 1b
Normalized moments of inertia, mean moment of inertia and derived C_{20} and C_{22}.

Reference	*A*	*B*	*C*	$M_{Ph}R_0^2$ (10^{24} kg m^{-2})	C_{20}	C_{22}
Bursa et al. (1990)	0.2787	0.3700	0.4026	1.73[a]	−0.0783	0.023
Borderies and Yoder (1990)	0.3347	0.3983	0.4671	1.40[b]	−0.1006	0.016
Willner et al. (2010)	0.3615	0.4265	0.5024	1.35[c]	−0.1084	0.016

[a] Bursa et al. (1990) used the GM_{Ph} value from Bills and Synnott (1987): $GM_{Ph} = 8.4 \times 10^{-4}$ km^3 s^{-2}.
[b] The GM_{Ph} from Berthias (1990) is used as the actual mass value at the time of the publication of Borderies and Yoder (1990).
[c] The MEX 2008 GM_{Ph} value from Andert et al. (2010) is used as the actual mass value at the time of publication of Willner et al. (2010).

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