Chinese Journal of Physics 56 (2018) 1525–1546

Contents lists available at ScienceDirect

Chinese Journal of Physics

journal homepage: www.elsevier.com/locate/cjph

# Entropy generation minimization and statistical declaration with probable error for skin friction coefficient and Nusselt number

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ARTICLE INFO

*Keywords:*
Two stretchable rotating disks
Porous disk
Thermal radiation
Joule heating
Viscous dissipation
Entropy generation
Probable error

ABSTRACT

Main emphasis of present work is to analyze the novel feature of entropy generation in MHD nanomaterial flow between two rotating disks. Heat transfer process is explored in the presence of Joule heating and thermal radiation. Tiwari–Das nanofluid model is employed in mathematical modeling. Aluminum oxide and copper water nanoparticles are accounted. Statistical declaration and probable error for problem accuracy are computed. Total entropy generation subject to Bejan number is scrutinized. Suitable variables are utilized to transform nonlinear PDEs to ordinary ones. Convergent series solutions are computed. Zeroth and *m*th order problems are discussed for stability analysis. The impact of physical flow variables like Reynolds number, magnetic parameter, porosity parameter, stretching parameter, rotational parameter, radiation parameter, Eckert number, suction injection parameter, Brinkman number and temperature ratio parameter on velocities, temperature, total entropy generation and Bejan number are examined and discussed through graphs. Velocity and thermal gradients at the surface of disks are computed.

## 1. Introduction

The second law of thermodynamics is extra trustworthy than the thermodynamics first law. It is because of the drawback of effectiveness of first law of thermodynamics in heat transport engineering systems. To minimizing the irreversibility one can use the second law of thermodynamics. Entropy generation is a principle of annihilation of the existing system. The assessment of entropy generation is implemented to enhance system performance. Joule heating, heat transfer, viscous dissipation, mass transfer etc. can be utilized as foundations of entropy generation. In recent years, numerous research articles have been published on the applications of entropy generation rates. Some recent studies on entropy generation can be seen in Refs. [1–10] and many studies therein.

Heat transport characteristics improvement through nanomaterials by rotating disks is an interesting topic of investigations these days. It is because of its huge ramifications in industrial and mechanical processes for example production process of plastic films, glass fiber innovation, glass fiber, wire drawing, melt spinning and some more. From extensive research, we concluded that nanofluids have higher thermal diffusivity than conventional base liquids. Thermal characteristics of conventional base fluids enhances when these nanoparticles are added in it. Word nanofluid was first used by Choi [11] and he proved that the addition of nanoparticles in the conventional liquids enhance the thermal attributes of these liquids. Further significant attempts on nanoliquid flows are seen in the investigations [12–23] and several studies therein.

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https://doi.org/10.1016/j.cjph.2018.06.023
Received 11 February 2018; Received in revised form 8 June 2018; Accepted 11 June 2018

**Nomenclature**

| | |
|---|---|
| $r, \vartheta, z$ | cylindrical coordinates [L] |
| $u, v, w$ | velocity components $[\frac{L}{T}]$ |
| $a_1, a_2$ | stretching rates of lower and upper disks $[\frac{1}{T}]$ |
| $\Omega_1, \Omega_2$ | rotational velocity of lower and upper disks $[\frac{1}{T}]$ |
| $T_1, T_2, T_f$ | temperatures of lower, upper disks and mean temperature [K] |
| $B_0$ | magnetic strength $[\frac{M}{T^2A}]$ |
| $\rho_{nf}$ | density $[\frac{M}{L^3}]$ |
| $\hat{p}$ | pressure $[\frac{M}{LT^2}]$ |
| $\nu_{nf}$ | kinematic viscosity $[\frac{L^2}{T}]$ |
| $\sigma$ | electrical conductivity $[\frac{A^2T^3}{ML^3}]$ |
| $k_{nf}$ | thermal conductivity $[\frac{ML}{T^3K}]$ |
| $c_p$ | specific heat $[\frac{L^2M}{T^2K}]$ |
| $W_0$ | suction/injection rate $[\frac{L}{T}]$ |
| $\sigma^\circ$ | Stefan–Boltzmann constant $[\frac{M}{T^3K^4}]$ |
| $R$, $Ec$ | radiation parameter, Eckert number |
| $k^\circ$ | mean absorption coefficient $[\frac{1}{L}]$ |
| $\theta, \phi$ | temperature and concentration profiles |
| $M$, $Pr$ | Hartmann number and Prandtl number |
| $W_s$, $Re$ | suction/injection parameter, Reynolds number |
| $A_1$ | stretching parameter of lower disk |
| $A_2$ | stretching parameter of lower disk |
| $\Omega$ | rotational parameter |
| $\alpha_1, \alpha_2$ | temperature ratio parameter |
| $Br$ | Brinkman number |

The investigation of heat transfer subject to porous space in view of its biological applications is an significant part of research. Such applications comprise liquid chromatography, tissue generation in scaffolds, blood flow through contracting muscle, transport of macromolecules in aortic media heat transfer in muscle and skin tissues and so forth. Recently some representative attempts for flows through porous space have been presented (see [24–30] and various studies therein).

Heat transport in the presence of rotating disk has gained much consideration in recent years. It is due to its both practical and theoretical values. Liquid flow by a rotating body has much significance in food processing technology, systems generating power, gas turbines, medical equipment, machines for air cleaning, electric aerodynamically engineering and food processing machines. The hydrodynamic flow due to an infinite rotating disk was discussed by Karman [31]. He established suitable transformations which have been used later to transform PDEs into ordinary one. Further some important investigations in this direction can be mentioned in refs. [32–40].

Present work aims to model three dimensional flow with entropy generation in MHD nanomaterial flow between two rotating disks. Heat transfer subject thermal radiation and Joule heating is discussed. Statistical declaration and probable error for problem accuracy are computed. Effects of thermal radiation and Joule heating are accounted. Total entropy generation subject to Bejan number is scrutinized. Suitable variables are utilized to transform nonlinear PDEs to ordinary ones and then solved by HAM [41–49]. Convergent series solutions are computed.

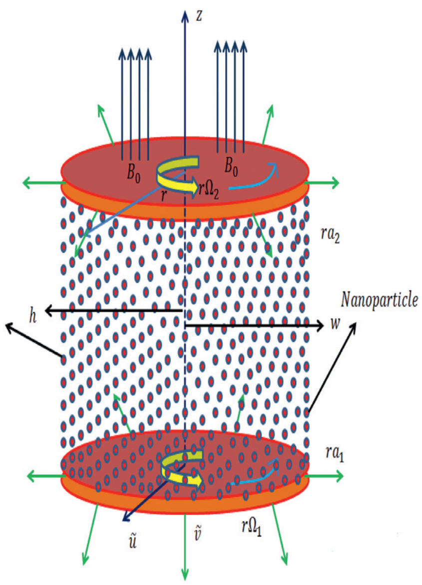


**Fig. 1.** Flow diagram.