Chinese Journal of Physics 56 (2018) 630–644

Contents lists available at ScienceDirect

Chinese Journal of Physics

journal homepage: www.elsevier.com/locate/cjph

Natural convection with damped thermal flux in a vertical circular cylinder

Najma Ahmed[a], Nehad Ali Shah[a], Dumitru Vieru[b,*]

[a] Abdus Salam School of Mathematical Sciences, GC University, Lahore, Pakistan
[b] Department of Theoretical Mechanics, Technical University of Iasi, Romania

ARTICLE INFO

Keywords:
Natural convection
Damped thermal flux
Caputo derivative
Analytical solutions

ABSTRACT

Natural convection flows of an incompressible Newtonian fluid inside a circular cylinder are studied. The heat transfer process is described by a generalized fractional constitutive equation for the thermal flux-temperature gradient. Caputo time-fractional derivative operator, which provides the damping of thermal flux, is considered into the studied model.

Analytical solutions to the fluid temperature, thermal flux, fluid velocity and volume flow rate are obtained with the integral transforms method (Laplace transform and finite Hankel transform).

Temperature behaviors for small and large values of the time *t*, as well as the post-transient and transient velocity components are determined. The influence of the memory parameter (the order of the time-fractional derivative) on the temperature, thermal flux, velocity and the volume flow rate is numerically and graphically studied.

1. Introduction

Among heat transfer problems, the natural convection heat transfer gains a special attention due to its wide applications in engineering systems. Free (natural) convection fluid flow is generated by the buoyancy due to the density variations resulting from temperature and/or concentration distribution. The free convection plays an important role in many engineering applications such as heat exchangers, drying processes, electrical components of transmission lines, nuclear energy fields, solar energy and thermal storage systems.

Heat transfer problems over/within vertical cylinders have wide range of applications in the field of technology. The chemical reactors and emergency cooling of nuclear fuel elements are important fields of heat transfer applicability. In polymer or glass industries, the hot filaments are considered as vertical cylinders which are cooled when they pass through the surrounding environment.

The natural convection phenomena in the case of vertical cylinders with basic equations to determine the heat transfer coefficient have been discussed by Cengel [1].

The steady, two-dimensional convective flow in a horizontal circular cylinder whose wall is non-uniformly heated was studied by Weinbaum [2]. He found that the flow which exhibits a nearly parabolic velocity profile to a critical Rayleigh number is gradually changed to a rigidly-rotating-core behavior as the Rayleigh number increases. Moukalled and Acharya [3] have numerically studied the natural convection heat transfer from a horizontal heated cylinder placed concentrically inside a square enclosure. An experimental study of natural convection flow through a vertical cylinder was made by Totala et al. [4]. They obtained the temperature

* Corresponding author.
E-mail address: dumitru_vieru@tuiasi.ro (D. Vieru).

https://doi.org/10.1016/j.cjph.2018.02.007
Received 19 November 2017; Received in revised form 12 February 2018; Accepted 12 February 2018
0577-9073/

variation along the length of cylinder, the average heat transfer coefficient and local heat transfer coefficient by using the energy balance. Paul and Deka [5] used the Laplace transform technique to study the unsteady one-dimensional free convection flow past an infinite vertical circular cylinder in a stratified fluid medium. Their results revealed that due to the effects of thermal and mass stratifications, fluid velocity and temperature had oscillatory behavior at small values of the time and reached steady state at larger values of time.

The effects of viscous dissipation, Joule heating and heat source/sink on magneto-hydrodynamic natural convection flows over a permeable horizontal circular cylinder in a porous medium was studied numerically by Prasad et al. [6]. Salman and Gheni [7] have investigated experimentally and numerically the natural convection heat transfer in an open inclined concentric annulus. The inner cylinder is unheated while the outer cylinder is subjected to a constant heat flux. They found that, the local Nusselt number and the average Nusselt number increase as the heat flux increases and when the inclination changes from horizontal position to vertical position. Nadeem et al. [8] obtained similarity solutions of the steady boundary layer flow and heat transfer of a second grade through a horizontal cylinder by using the homotopy analysis method. An experimental study for the convective heat transfer coefficient inside a rotating cylinder with an axial airflow was conducted by Seghir-Ouali et al. [9]. Effects of Soret number, Dufour number and order of chemical reaction on the separation of a binary fluid mixture confined between two concentric stationary circular cylinders embedded in a porous medium in the presence of uniform radial magnetic field were numerically investigated by Sharma and Borgohain [10].

In the last years, the theory of fractional calculus (the integrals and derivatives of non-integer order) has gained considerable interest due to its applications in different areas including mathematical biology, thermo-elasticity, electromagnetic waves in dielectric media, flow of complex rheological media and fluids, reaction–diffusion equation [11]. Fractional derivative operators in the sense of the Riemann–Liouville, Caputo–Liouville or Caputo–Fabrizio definitions were successfully used in the anomalous diffusion models in complex media [12–14]. A very instructive presentation of the applications of fractional derivatives in transport processes is made by Hristov [15].

Baleanu et al. [16] using the variational homotopic perturbation method and q-homotopic analysis method have studied the fractional advection partial differential equation modeled by the time-fractional Caputo derivative and time-fractional Caputo–-Fabrizio derivative. The spatial behavior of the harmonic in time vibrations in the model of linear thermo-elasticity theory without dissipation of energy for micropolar bodies was investigated by Marin and Baleanu [17].

Yasir et al. [18] studied the unsteady flows of upper-convected Maxwell fluids, in two-dimensional boundary layer approximation using a fractional model based on the time-fractional Caputo–Fabrizio derivatives. They determined analytical solutions for velocity components by using the generalized method of separation of variables coupled with the Laplace transform method.

Unsteady rotational flows of the fractional fluid with two thermo-mechanical parameters of Xanthan gum between two cylinders, in the presence of a transverse magnetic field, were studied by Hamza et al. [19] by using the Laplace transform method and numerical algorithms for the inversion of Laplace transforms.

Raza et al. [20] investigated the unsteady flow of an Oldroyd-B fluid with Caputo fractional derivatives through an infinite oscillating cylinder. The heat transfer analysis on peristaltic transport of ree-Eyring fluid in rotating frame was investigated by Hayat et al. [21]. Guo and Qi [22] investigated electro-osmotic peristaltic flows of the fractional Jeffrey's viscoelastic fluid through a cylindrical micro-channel under the assumptions of long wavelength, low Reynolds number and Debye–Hückel linearization. The transient electroosmotic flow of fractional Maxwell fluids in a rectangular microchannel was studied by Yang et al. [23].

In this paper the natural convection flow of an incompressible Newtonian fluid through a vertical cylinder is studied by considering the heat transfer process described with a generalized fractional constitutive equation for the thermal flux-temperature gradient. In order to develop this mathematical model, the Caputo time-fractional derivative operator is employed. This model provides a damping of the thermal flux.

Analytical solutions for the fluid temperature, thermal flux, fluid velocity and volume flow rate are obtained with the integral transforms method (Laplace transform and finite Hankel transform). The general solutions are expressed with Mittag–Leffler functions of one-parameter. In order to simplify the numerical calculations, the integral representation of the Mitag–Leffler functions is used.

By using the asymptotic representation of the Mittag–Lefler functions, the temperature behaviors for small and large values of time are obtained. In addition, the post-transient and transient velocity components are also determined.

The influence of the memory parameter (the order of the time-fractional derivative) on the temperature, thermal flux, velocity and the volume flow rate is numerically and graphically studied.

It is found that the heat transfer process and the fluid flow are different at small and large values of time. This behavior is closely related to the kernel of the Caputo derivative that provides the weight function of the temperature gradient. Based on the properties of the power-law Caputo kernel, which becomes the Dirac distribution when the fractional parameter tends to 1, the solutions for ordinary case are obtained as a limit case of the general solutions.

2. Mathematical formulation of the problem

The transient free convection flow of an incompressible, linear-viscous fluid in an infinite vertical circular cylinder of radius r_0 is considered. The thermal transport process is described by the Caputo time-fractional derivative with weakly singular kernel. A generalized fractional thermal constitutive equation which provides a weighted average of temperature gradient over the past history is used.

The z-axis is considered along the axis of cylinder in vertical upward direction and the radial coordinate r is taken normal to the