Chinese Journal of Physics 56 (2018) 91–95

Contents lists available at ScienceDirect

Chinese Journal of Physics

journal homepage: www.elsevier.com/locate/cjph

Inflationary magnetic fields from massive Euler–Heisenberg electrodynamics

Leonardo Campanelli

All Saints University, Asudom Academy of Science, 5145 Steeles Ave. W., Toronto, Ontario, Canada

A B S T R A C T

We study the generation of cosmic magnetic fields during de Sitter inflation in a non-conformally-invariant effective model of massive electrodynamics containing a four-photon interaction term. We show that, if the photon self-coupling is strong enough, comoving magnetic fields correlated on scales of 10kpc and of intensities 10^{-22} G$\lesssim B_0 \lesssim 10^{-19}$ G are produced as excitation of the vacuum. If amplified by galactic dynamo, they naturally explain the existence of galactic magnetic fields.

1. Introduction

Coherent magnetic fields as strong as $B \sim 10^{-6}$ G have been detected in any type of galaxies [1]. Their origin is still an open issue and is puzzling to the point that "galactic magnetism" can be considered one of the biggest mysteries in cosmology.

Nowadays, what it clear enough is that, if seed magnetic fields are present prior to galaxy formation, due to protogalaxy collapses and magnetohydrodynamic turbulence effects, they can be amplified and then, at least in principle, reproduce the properties of presently-observed galactic fields. However, this process –known as "dynamo mechanism"–, is successful provided that the intensity and correlation length of the seed field are $B \gtrsim 10^{-33}$ G and $\lambda \gtrsim 10$ kpc [2]. If the galactic dynamo is inefficient, instead, a comoving seed field $B \gtrsim 10^{-14}$ G correlated today on scales of order 1 Mpc is needed to explain the existence of galactic fields [2].

A plethora of mechanisms have been proposed to produce seed magnetic fields since Fermi's proposal of existence of cosmic magnetic fields back in 1949 [3]. Promising candidates for the generation of seed fields are those mechanism operating during inflation since in this case the generated fields can be correlated on super-horizon scales, and then their comoving correlation length can be as large as the galactic one (see, e.g., [4–7]). If magnetic fields are created after inflation, instead, their correlation length cannot exceed the dimension of the horizon, so that they are correlated on scales generally much smaller than the characteristic scale of the observed cosmic fields [however, if magnetohydrodynamic turbulence operates during their evolution, an enhancement of correlation length can occur (see, e.g., [8,9])]. Since the standard (Maxwell) electrodynamics is conformally invariant, magnetic fields generated at inflation are vanishingly small. For this reason, all generating models proposed in the literature repose on the breaking of conformal invariance of Maxwell theory.

In this paper, we study the possibility to generate seed magnetic fields during inflation in a non-conformally invariant theory of electromagnetism described by the Lagrangian

$$\mathscr{L} = -F + \frac{1}{2} m_\gamma^2 A_\mu A^\mu + \frac{1}{\mu^4} F^2, \tag{1}$$

where $F = \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field strength tensor. The second term in the above Lagrangian

E-mail address: leonardo.s.campanelli@gmail.com.

https://doi.org/10.1016/j.cjph.2017.12.007
Received 18 November 2017; Accepted 5 December 2017
Available online 07 December 2017

allows the photon to have an (effective) mass m_γ, while the last term describes a four photon interaction, μ being a parameter having dimension of a mass. A Lagrangian of this type can naturally arise as an effective (Euler–Heisenberg) Lagrangian induced by a charged scalar field minimally coupled to the photon.[1] In this paper, however, we do not consider any particular mechanism leading to Lagrangian (1) and we assume, then, that m_γ and μ are free parameters.

In the next Section, we will show that, if the photon self-coupling is strong enough, magnetic fields produced in this model have the right intensity and correlation length to trigger the dynamo mechanism, and then to explain galactic magnetism.

2. Magnetic fields from inflation

We assume that during inflation the Universe is described by a de Sitter metric $ds^2 = a^2(d\eta^2 - d\mathbf{x}^2)$, where $a(\eta)$ is the expansion parameter, $\eta = -1/(aH)$ is the conformal time, and H is the Hubble parameter. The conformal time is related to the cosmic time t through $d\eta = dt/a$. We normalize the expansion parameter so that at the present time t_0, $a(t_0) = 1$.

In general, we can decompose the electromagnetic field A_ν in a transverse part $\mathbf{A}_T$ and a longitudinal part $\mathbf{A}_L$, $A_\mu = (A_0, \mathbf{A}_T + \mathbf{A}_L)$. Here, A_0 is a non-dynamical degree of freedom and can be expressed in terms of $\mathbf{A}_L$. Lagrangian (1) gives equations of motion for the transverse and longitudinal parts of the photon field,

$$\mathscr{L}_F \ddot{\mathbf{A}}_T + \dot{\mathscr{L}}_F \dot{\mathbf{A}}_T + a^2 m_\gamma^2 \mathscr{L}_F \mathbf{A}_T = \mathscr{L}_F \nabla^2 \mathbf{A}_T - \nabla \mathscr{L}_F \times (\nabla \times \mathbf{A}_T), \tag{2}$$

and

$$\mathscr{L}_F \ddot{\mathbf{A}}_L + \dot{\mathscr{L}}_F \dot{\mathbf{A}}_L + a^2 m_\gamma^2 \mathscr{L}_F \mathbf{A}_L = \mathscr{L}_F \nabla \dot{A}^0 + \dot{\mathscr{L}}_F \nabla A^0, \tag{3}$$

respectively, where a dot denotes differentiation with respect to η. Here,

$$\mathscr{L}_F = -1 + \frac{2F}{\mu^4} \tag{4}$$

is the derivative of the Lagrangian (1) with respect to F, and

$$F = \frac{1}{2a^4}[(\nabla \times \mathbf{A}_T)^2 - \dot{\mathbf{A}}_T^2 - (\dot{\mathbf{A}}_L - \nabla A^0)^2]. \tag{5}$$

We assume that the effective Lagrangian (1) reduces to the Maxwell one, $\mathscr{L} = -F$, in the limit of Minkowskian spacetime ($\eta \to -\infty$) and after inflation. In this case, a possible solution of the equations of motion is such that

$$\mathbf{A}_L = 0, \quad A^0 = 0, \tag{6}$$

owing to the fact that Eq. (3) is homogeneous with respect to $\mathbf{A}_L$ and A_0 and at the initial time, $\eta \to -\infty$, we can set $\mathbf{A}_L = 0$ and $A_0 = 0$. Consequently, the only remaining degree of freedom of the theory is the transverse part of the vector potential.

We are interested in the study of large-scale electromagnetic fields, that is in modes whose physical wavelength is much greater then the Hubble radius, $\lambda_{\text{phys}} \gg H^{-1}$ or equivalently $-k\eta \ll 1$, where $\lambda_{\text{phys}} = a\lambda$ and $\lambda = 1/k$ is the comoving wavelength. For these modes, we can neglect the right-hand side of Eq. (2) since $|\mathscr{L}_F \nabla^2 \mathbf{A}_T|/|\mathscr{L}_F \ddot{\mathbf{A}}_T| \sim |\nabla \mathscr{L}_F \times (\nabla \times \mathbf{A}_T)|/|\mathscr{L}_F \ddot{\mathbf{A}}_T| \sim (k\eta)^2$. Also, since $|\nabla \times \mathbf{A}_T|/|\dot{\mathbf{A}}_T| \sim |k\eta|$, we can write $F \simeq -\dot{\mathbf{A}}_T^2/2a^4$.[2] Moreover, we assume that the field is "strong", in the sense that $|F| \gg \mu^4/2$. In this case, we have $\mathscr{L}_F \simeq 2F/\mu^4$. Accordingly, the evolution equation for the large-scale modes of the transverse part of the electromagnetic field becomes

$$\dot{\mathbf{A}}_T^2 \ddot{\mathbf{A}}_T + 2\left(\dot{\mathbf{A}}_T \ddot{\mathbf{A}}_T + \frac{2}{\eta} \dot{\mathbf{A}}_T^2\right) \dot{\mathbf{A}}_T + \frac{m_\gamma^2}{H^2\eta^2} \dot{\mathbf{A}}_T^2 \mathbf{A}_T = 0. \tag{7}$$

Multiplying both sides of the above equation by $\dot{\mathbf{A}}_T$ and dividing by $\dot{\mathbf{A}}_T^2$ afterwards, we obtain

$$3\dot{\mathbf{A}}_T \ddot{\mathbf{A}}_T + \frac{4}{\eta} \dot{\mathbf{A}}_T^2 + \frac{m_\gamma^2}{H^2\eta^2} \mathbf{A}_T \dot{\mathbf{A}}_T = 0. \tag{8}$$

To proceed further, we expand $\mathbf{A}_T$ as

$$\mathbf{A}_T = \int \frac{d^3k}{(2\pi)^3\sqrt{2k}} \sum_{\alpha=1,2} \boldsymbol{\varepsilon}_{\mathbf{k},\alpha}\, a_{\mathbf{k},\alpha} A_k(\eta)\, e^{i\mathbf{k}\mathbf{x}} + \text{h.c.}, \tag{9}$$

where $k = |\mathbf{k}|$ and $\boldsymbol{\varepsilon}_{\mathbf{k},\alpha}$ are the transverse polarization vectors satisfying the completeness relation

$$\sum_\alpha (\varepsilon_{\mathbf{k},\alpha})_i (\varepsilon^*_{\mathbf{k},\alpha})_j = \delta_{ij} - \frac{k_i k_j}{k^2}, \tag{10}$$

[1] The Euler–Heisenberg Lagrangian is $\mathscr{L}_{EH} = -F + F^2/\mu^4 + G^2/\mu'^4$, where $G = \frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}$, $\tilde{F}^{\mu\nu}$ is the dual electromagnetic field strength tensor, and μ' a parameter having dimension of a mass. In this paper, we are concerned only with a Lagrangian depending on the invariant F (see footnote 2 for a justification).

[2] We did not consider terms proportional to G^2 in Lagrangian (1) because they are negligible with respect to the F^2-terms on large scales. In fact, since $G = -(\dot{\mathbf{A}}_T \nabla \times \mathbf{A}_T)/a^4$, we have $|G^2|/|F^2| \sim (k\eta)^2$.